According to New Syllabus

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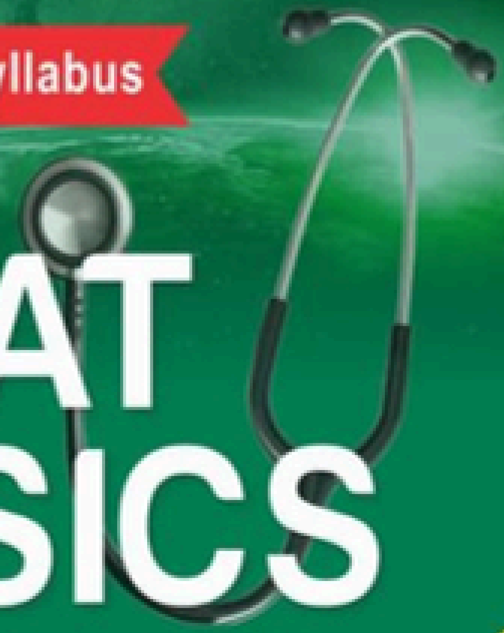
MDCAT PHYSICS

PREPARATION & PRACTICE BOOK



REVISED EDITION

SELF STUDY GUIDE

TO SECURE 100% SUCCESS

Salient Features

- According to revised syllabus
- A strong emphasis on conceptual understanding
- Short cuts trick and techniques to solve problems quickly
- Urdu Explanation of Difficult Concepts
- A concise summary of formulas at the end of each chapter
- Practice book containing 2500 plus MCQ's
- Solution of difficult MCQ'S

Written By:

AZHAR IQBAL

FAISAL NADEEM

Entry Test Preparation & Practice Book

A solution to

MDCAT PHYSICS

Self Study

Features

- Easy and Concise material
- Short Tricks to solve MCQs
- Urdu Explanation is added for easy understanding
- Key and explanation of MCQs are included

Written by

Prof. AZHAR IQBAL

MOB: 0336-7098894

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1st Edition

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PREFACE

I, **PROF. AZHAR IQBAL**, am teaching Physics to the students of MDCAT and F.Sc from the last 12 years. Since long, I have been observing the problems of Students of MDCAT, as well as feeling that for the preparation of MDCAT, there are no such books in the market that can solve the problems of the students. Therefore, in order to solve the problems of the students of MDCAT, I decided to write comprehensive book.

Now, by the grace of Allah, The Almighty, I have successfully written a book for the student of MDCAT. The book contains two portions i.e. MDCAT preparation book and MDCAT practice book. This book is a combination multiple extraordinary qualities that have never been observed in any other book.

- All the topics have been written in quite an easy way.
- The book has been written according to prescribed syllabus of MDCAT
- Difficult questions have solved by using short tricks
- A chapter consisting of short tricks have also been added in the book
- Difficult concepts have also been described in Urdu for the convenience and better comprehension of the students.

I hope this book will be quite helpful in solving the problems of the students of MDCAT. It will lead them to the target learning as well.

I am looking forward for your valuable suggestions and feedback. So that I will make improvements in this effort in my next edition.

Best wishes

Author
Prof. AZHAR IQBAL
WhatsApp: 0336 709 8894

DEDICATION

To

My Beloved Wife,

For her continued unfailing Love, Support and Understanding that made the completion of this book possible.

(Preparation Book)

CONTENTS

(Practice Book)

CONTENTS

PREPARATION BOOK

UNIT 00 ›› BASIC TRICKS

Important Tips to Solve Physics MCQ's

1. فزکس کے زیادہ تر MCQ's کا تعلق Formulas کے ساتھ ہوتا ہے اس لئے جب بھی آپ کوئی Topic پڑھیں تو اس میں استعمال ہونے والے Formulas کو اچھی طرح یاد کریں اور Formulas سے بننے والے مندرجہ ذیل قسم کے سوالات کو ضرور مدِ نظر رکھیں۔

1. ایک Quantity کے بڑھنے یا کم ہونے سے کسی دوسری Quantity کی value پر کیا اثر ہوگا۔

Example:
By increasing the distance from the positive point charge its potential:
(a) Increases (b) Decreases ✓
(c) remain same (d) either increases or decreases

Solution:
As $V = \frac{Kq}{r}$ or $V \propto \frac{1}{r}$
By increasing distance V decreases.

2. ایک Quantity کی values میں Ratio دی گئی ہو تو دوسری Quantity کی values میں Ratio کیا ہوگی۔

Example:
Two bodies having equal mass are moving with velocities $10\ ms^{-1}$ and $20\ ms^{-1}$ then the ratio between their K.E is
(a) 1 : 2 (b) 2 : 1 (c) 1 : 4✓ (d) 4 : 1

Solution:
As $K.E = \frac{1}{2}mv^2$ or $K.E \propto v^2$
The ratio between velocity is 1 : 2 so ratio in K.E is 1 : 4

3. اگر کسی ایک Quantity کو Double یا Half وغیرہ کر دیا جائے تو دوسری Quantity پر کیا اثر ہوگا۔

Example:
If length of pendulum is doubled then its time period will become
(a) Double (b) Half (c) $\sqrt{2}$ times✓ (d) $\frac{1}{\sqrt{2}}$ times

Solution: As $T = 2\pi\sqrt{\frac{\ell}{g}}$ or $T \propto \sqrt{\ell}$ If length is doubled T becomes $\sqrt{2}$ times.

(Two-Two Values Relationship)

4. اگر ایک Quantity کی دو ویلیوز اور دوسری Quantity کی ایک value دی گئی ہو تو اس کی دوسری ویلیو کیا ہوگی۔

Example:
If at pressure 10 atm the volume of the gas is $2m^3$. At what pressure the volume of the gas will be $5m^3$.
(a) 5 atm (b) 4 atm✓ (c) 3 atm (d) 2 atm

Solution:
$\frac{P_2}{P_1} = \frac{V_1}{V_2} \Rightarrow P_2 = \frac{P_1 V_1}{V_2} = \frac{10 \times 2}{5}$
$= 4\ atm$

5. ایک Quantity کی دوسری Quantities سے dependent یا independent ہے۔

Example:
Speed of sound in air is independent of :
(a) density of air (b) pressure of air✓
(c) Temperature of air (d) None of these

Solution:
As $v = \sqrt{\frac{\gamma P}{\rho}}$ and $\rho \propto P$

(Numerical Type Question)

6. Numerical Type Question کو solve کرنے کے لیے question میں دیکھیں کہ کونسی Quantities دی گئی ہیں اور کس کے بارے میں پوچھا گیا ہے اور ان کا آپس میں Relation کیا ہے۔

Example: If a body of a mass 2kg is moving with momentum 6Ns then its K.E will be:
(a) 6 J (b) 9 J✓ (c) 12 J (d) 15 J

Solution: As $K.E = \frac{p^2}{2m} = \frac{6^2}{2\times2} = \frac{36}{4} = 9J$

(Re-arranging Formulas)

7. کسی بھی فارمولے کو Re-arrange کر کے یا Variables کا باہم بدل کر پوچھا جا سکتا ہے۔

Example:
If T is time period of simple pendulum at a place where acceleration due to gravity is g then length of pendulum is:
(a) $\frac{gT^2}{4\pi^2}$ ✓ (b) $\frac{gT}{4\pi^2}$ (c) $\frac{4\pi^2}{gT^2}$ (d) $\frac{4\pi}{gT^2}$

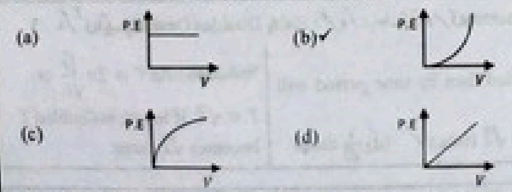
Solution:
As $T = 2\pi\sqrt{\frac{\ell}{g}}$
$\ell = \frac{gT^2}{4\pi^2}$

(Graphical Types Question)

8. Quantities میں گراف کی باہم گراف کی slope کیا بتاتی ہے گراف کے نیچے Area کیا بتاتا ہے۔

Example 1: The graph between potential energy stored in a capacitor verses voltage across the capacitor is:

(a) (b)✓ (c) (d)



Solution 1:
As $P.E = \frac{1}{2}CV^2$
$P.E \propto V^2$
So the graph between P.E and Voltage is a parabola.

Example 2: The slope of velocity time graph represent the:
(a) displacement (b) acceleration✓ (c) momentum (d) force

Solution 2:
$Slope = v/t = a$

Example 3: Area under force displacement graph represent the:
(a) acceleration of the body (b) Work done on the body✓
(c) Power of the body (d) Impulse on the body

Solution 3:
$area = Fd = W$

2. Important Relations Between Quantities

Relation	If value of x increases then y	If value of x doubled then y	If values of x are in ratio 2:3 then ratio in y is	Two-Two values relation
$y \propto x$ Example: $Q = CV$ $\Rightarrow Q \propto V$	Increases	**Doubled** جتنا x میں change آئے گا Y بھی اتنے time ہی change ہوگا	**2 : 3** جو x میں ratio ہوگی وہی y میں بھی ہوگی	$\frac{y_2}{y_1} = \frac{x_2}{x_1}$
$y \propto x^2$ Example: $K.E = \frac{1}{2}mv^2$ $\Rightarrow K.E \propto v^2$	Increases	**Four times** x کی values کا square کرنے سے y میں change آ جائے گا	x کی values کا square کرنے سے y میں ratio آ جائے گی	$\frac{y_2}{y_1} = \frac{x_2^2}{x_1^2}$
$y \propto \sqrt{x}$ Example: $T = 2\pi\sqrt{\frac{\ell}{g}}$ $\Rightarrow T \propto \sqrt{\ell}$	Increases	**$\sqrt{2}$ times** x کی values کا $\sqrt{\ }$ کرنے سے y میں change آ جائے گا	**$\sqrt{2} : \sqrt{3}$** x کی values کا $\sqrt{\ }$ کرنے سے y میں ratio آ جائے گی	$\frac{y_2}{y_1} = \frac{\sqrt{x_2}}{\sqrt{x_1}}$
$y \propto \frac{1}{x}$ Example: $PV = Const.$ $\Rightarrow P \propto \frac{1}{V}$	Decreases	**Halved** جتنا x میں change آئے گا Y میں اتنے time الٹ change آئے گا	**3 : 2** جو x میں ratio ہوگی y میں اس کے الٹ ratio ہوگی	$\frac{y_2}{y_1} = \frac{x_1}{x_2}$
$y \propto \frac{1}{x^2}$ Example: $E = \frac{kq}{r^2}$ $\Rightarrow E \propto \frac{1}{r^2}$	Decreases	**$\frac{1}{4}$ times** x کی values کا square کرنے اور الٹا کرنے سے y میں change آ جائے گا	**9 : 4** x کی values کا square کرنے اور الٹا کرنے سے y میں ratio آ جائے گی	$\frac{y_2}{y_1} = \frac{x_1^2}{x_2^2}$
$y \propto \frac{1}{\sqrt{x}}$ Example: $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ $\Rightarrow f \propto \frac{1}{\sqrt{m}}$	Decreases	**$\frac{1}{\sqrt{2}}$ times** x کی values کا $\sqrt{\ }$ کرنے اور الٹا کرنے سے y میں change آ جائے گا	**$\sqrt{3} : \sqrt{2}$** x کی values کا $\sqrt{\ }$ کرنے اور الٹا کرنے سے y میں ratio آ جائے گی	$\frac{y_2}{y_1} = \frac{\sqrt{x_1}}{\sqrt{x_2}}$

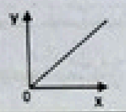
نوٹ: کوئی بھی دو quantities میں Proportionality relation کے لیے باقی quantities کا constant ہونا ضروری ہے۔

3. How to Determine Graph?

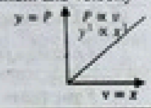
(i) $y^1 \propto x^1$

اگر x اور y دونوں کی پاورز Relation میں one ہو تو گراف Straight line ہوگا اور origin سے پاس کرے گا

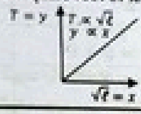
Graph:



Example 1: Graph b/w momentum and velocity



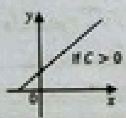
Example 2: Graph b/w time period and square root of length



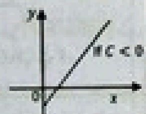
(ii) $y^1 = mx^1 + c$

اگر x اور y دونوں کی پاورز one ہو لیکن کوئی constant بھی add ہو تو گراف origin سے پاس نہیں کرے گا

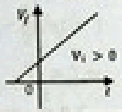
Graph:



Graph:



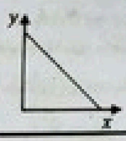
Example 2: $v_f = v_i + at^1$



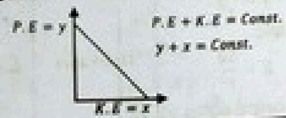
(iii) $x^1 + y^1 = Constant$

اگر x اور y دونوں کی پاورز one ہو لیکن ان کا sum کانسٹنٹ ہو تو گراف Straight line ہوگا

Graph:



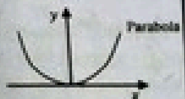
Example 2:



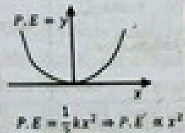
(iv) $y^1 \propto x^2$

اگر ایک variable کی پاور one اور دوسرے کی پاور two ہو تو گراف Parabola ہوگا۔

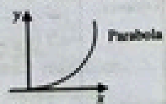
Graph:



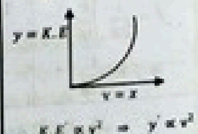
Example:



For 1st Quadrant



Example:

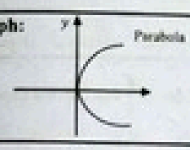


نوٹ: جس variable کی پاور کم ہو گی گراف اسی کی طرف bend ہوگا۔

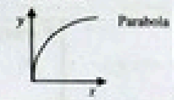
(v) $y \propto \sqrt{x}$

اگر ایک variable کی پاور one اور دوسرے پر √ ہو تو گراف Parabola ہوگا۔

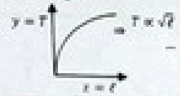
Graph:



For 1st Quadrant



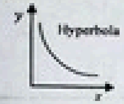
Example:



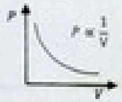
(vi) $y \propto \frac{1}{x}$

اگر دونوں variables ایک دوسرے کے Inversely Proportional ہوں تو گراف Hyperbola ہوگا۔

Graph:



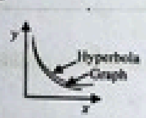
Example : For Boyle's Law



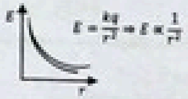
(vii) $y \propto \frac{1}{x^2}$

اگر ایک variable دوسرے کے Square کے Inversely Proportional ہوں تو گراف Hyperbola سے زیادہ Steeper ہوگا۔

Graph:



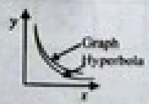
Example :



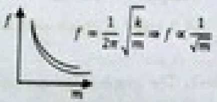
(viii) $y \propto \frac{1}{\sqrt{x}}$

اگر ایک variable دوسرے کے √ کے Inversely Proportional ہوں تو گراف Hyperbola سے کم Steeper ہوگا۔

Graph:



Example :

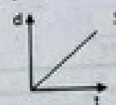


4. Slope of the Graph:

$Slope = \frac{y}{x}$

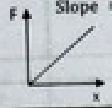
گراف کی slope اس quantity کو represent کرتی ہے جو کہ y والی quantity کو x-axis والی quantity سے divide کرنے سے ملے

Example:

(i) 

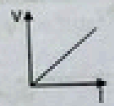
Slope $= \frac{d}{t} = v$ (velocity)

Example:

(ii) 

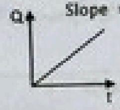
Slope $= \frac{F}{x} = k$ (Spring Constant)

Example:

(iii) 

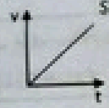
Slope $= \frac{V}{I} = R$(Resistance)

Example:

(iv) 

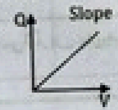
Slope $= \frac{Q}{t} = I$ (Current)

Example:

(v) 

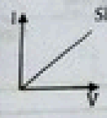
Slope $= \frac{v}{t} = a$ (acceleration)

Example:

(vi) 

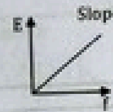
Slope $= \frac{Q}{V} = C$ (Capacitance)

Example:

(vii) 

Slope $= \frac{I}{V} = G$ (Conductance)

Example:

(viii) 

Slope $= \frac{E}{f} = h$ (Plank's Const.)

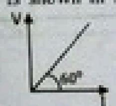
5. How to Find the Slope?

(i). $Slope = Tan\theta$

(θ is angle of graph with x-axis)

θ	0°	30°	45°	60°	90°
$Tan\,\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Example-1: The graph between voltage and current in SI unit is shown in figure below then resistance will be:



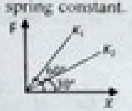
(a) $1/\sqrt{3}$ (b) $\sqrt{3}$ ✓ (c) 1 (d) 0.5

Solution:

$R = \frac{V}{I} = slope$

$= Tan\,60° = \sqrt{3}$

Example-2: The graph between force and extension for two spring is shown in figure below then find the ratio between the spring constant.



(a) 1 : 3 (b) 3 : 1 ✓ (c) 1 : 1 (d) 9 : 1

Solution: As

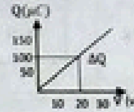
$K = \frac{F}{x} = slope = Tan\theta$

$\frac{K_1}{K_2} = \frac{Tan\,60°}{Tan\,30°}$

$= \frac{\sqrt{3}}{1/\sqrt{3}} = \frac{3}{1}$

(ii). $Slope = \frac{\Delta y}{\Delta x}$

Example: The graph between charge and voltage for capacitor is shown in figure below. The capacitance of capacitor will be:



(a) $5\mu F$ ✓ (b) $2.5\mu F$ (c) $15\mu F$ (d) $10\mu F$

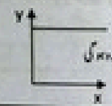
Solution:

$C = \frac{\Delta y}{\Delta x}$

$= \frac{\Delta Q}{\Delta V} = \frac{100\mu c}{20V}$

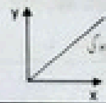
$= 5\mu F$

DIFFERENT CASES FOR SLOPE



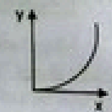
slope = 0

اگر گراف Horizontal line ہو تو slope صفر ہوگی



slope = const.

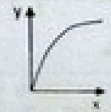
اگر گراف straight line ہو تو slope مستقل ہوگی



Slope is increasing.

اگر گراف y-axis کی طرف bend ہو تو

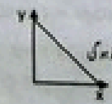
slope بڑھ رہی ہے



Slope is decreasing.

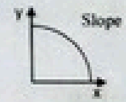
اگر گراف x-axis کی طرف bend ہو تو

slope کم ہو رہی ہے



Slope is negative and const.

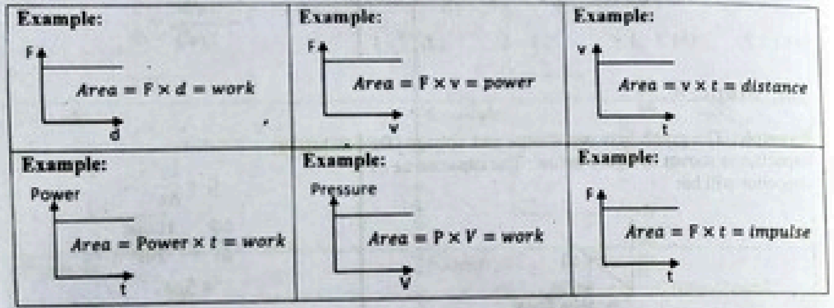
اگر گراف downward ہو تو slope منفی ہوگی



Slope is negative and increasing

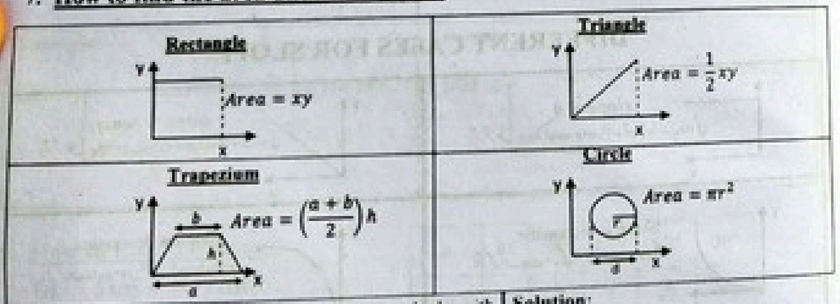
6. Area Under the Graph:

گراف کے Area کی quantity کو represent کرتا ہے جو کہ x-axis اور y-axis کی quantities کو multiply کرنے سے، دیکھیں کہ کونسی quantity حاصل ہوتی ہے۔

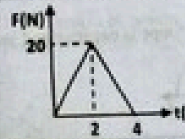


Note: Greater the area under the graph, larger the quantity.

7. How to find the area under the graph.



Example: The variation of force acting on a body with time is shown. What is the change in momentum of body after 4 s?



(a) 10 Ns (b) 20 Ns (c) 40 Ns ✓ (d) 80 Ns

Solution:

$$Change\ in\ momentum = impulse$$
$$= Area\ of\ triangle$$
$$= \frac{1}{2} base \times height$$
$$= \frac{1}{2} \times 4 \times 20 = 40\ Ns$$

8. Percentage Type Questions:

1st Type:

If percentage change in X is less than 10% then percentage change in Y is given as:

$$\%\ change\ in\ Y = Power\ of\ X\ (\%\ change\ in\ X)$$

Example:

If length of pendulum is increased by 6% then its time period will be increased by:

(a) 6% (b) $\sqrt{6}$% (c) 3% ✓ (d) 1.5%

Solution: As $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T \propto \sqrt{\ell}$

$$\%change\ in\ T = \frac{1}{2}(\%change\ in\ \ell) = 3\%$$

2nd Type **Suppose X and Y are related as $Y \propto X^2$**

If percentage change in X is greater than 10% then percentage change in Y is given as:

(i). When X increases:

$$\%\ change\ in\ Y = 2(\%\ change\ in\ X) + \left(\frac{\%\ change\ in\ X}{10}\right)^2$$

Example:

If momentum of a body increases by 20% then its K.E will be increased by:

(a) 20% (b) 40% (c) 44% ✓ (d) 400%

Solution: As $K.E = \frac{P^2}{2m} \Rightarrow K.E \propto P^2$

$$\%\ change\ in\ K.E = 2(20\%) + \left(\frac{20\%}{10}\right)^2 = 44\%$$

(ii). When X decreases:

$$\%\ change\ in\ Y = 2(\%\ change\ in\ X) - \left(\frac{\%\ change\ in\ X}{10}\right)^2$$

Example:

If momentum of a body decreases by 10% then its K.E will be decreased by:

(a) 9% (b) 5% (c) 10% (d) 19% ✓

Solution: As $K.E = \frac{P^2}{2m} \Rightarrow K.E \propto P^2$

$$\%\ change\ in\ K.E = 2(10\%) - \left(\frac{10\%}{10}\right)^2 = 44\%$$

3rd Type: **If change in X is multiple of 100%.**

Example:

If length of the pendulum is increased by 100% then its time period will become:

(a) Double (b) Half (c) $\frac{1}{\sqrt{2}}$ times (d) $\sqrt{2}$ times ✓

Solution: As $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T \propto \sqrt{\ell}$

If length increases by 100% its mean it is doubled. Then time period will become $\sqrt{2}$ times.

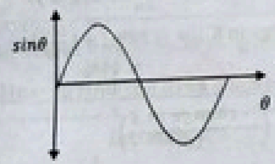
% Change	100 %	200 %	300 %	400 %
Quantity Becomes	Doubled	Three times	Four times	Five times

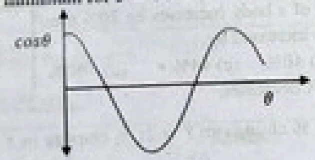
9. Trigonometric Ratios.

θ	0°	30°	45°	60°	90°
$sin\theta$	0 0%	$\frac{1}{2} = 0.5 = 50\%$	$\frac{1}{\sqrt{2}} = 0.7 = 70\%$	$\frac{\sqrt{3}}{2} = 0.86$ $= 86\%$	1 100%
$cos\theta$	1% 100%	$\frac{\sqrt{3}}{2} = 0.86 = 86\%$	$\frac{1}{\sqrt{2}} = 0.7 = 70\%$	$\frac{1}{2} = 0.5$ $= 50\%$	0 0%

Notice from the above table:

- Value of $sin\theta$ increases by increasing angle and vice versa.
- Value of $cos\theta$ decreases by increasing angle and vice versa.
- $sin\theta$ is maximum for $\theta = 90°$ and minimum for $\theta = 0°$.
- $cos\theta$ is maximum for $\theta = 0°$ and minimum for $\theta = 90°$.





PREFIXES

Prefix	Decimal	Multiplier	Prefix	Decimal	Multiplier
Yotta	10^{24}	Y	Deci	10^{-1}	d
Zetta	10^{21}	Z	Centi	10^{-2}	c
Exa	10^{18}	E	Milli	10^{-3}	m
Peta	10^{15}	P	Micro	10^{-6}	μ
Tera	10^{12}	T	Nano	10^{-9}	n
Giga	10^{9}	G	Pico	10^{-12}	p
Mega	10^{6}	M	Femto	10^{-15}	f
Kilo	10^{3}	k	Atto	10^{-18}	a
Hecto	10^{2}	h	Zepto	10^{-21}	z
Deca	10^{1}	da	Yocto	10^{-24}	y

SI BASE QUANTITIES AND BASE UNITS

Base quantity		SI Base Unit	
Name	Symbol	Name	Symbol
Length	L	Meter	m
Mass	m	Kilogram	kg
Time, duration	t	Second	s
Electric current	I	Ampere	A
Thermodynamic temperature	T	Kelvin	K
Amount of substance	n	Mole	mol
Luminous intensity	I_v	Candela	cd

SOME IMPORTANT CGS UNIT AND THEIR RELATION WITH SI UNIT

Quantity	cgs units	Equivalent in SI units
Length	cm(centimeter)	$10^{-2}m$
Mass	g (gram)	$10^{-3}kg$
Time	s (second)	s
Force	dyn (dyne)	$10^{-5}N$
Energy	erg	$10^{-7}J$
Charge	(Franklin) Fr	$3.34 \times 10^{-10}C$
Current	Bi (biot)	$10A$
Heat energy	Cal (calorie)	$4.18\,J$
Magnetic flux density	G (gauss)	$10^{-4}T$
Magnetic flux	Mx (Maxwell)	$10^{-8}Wb$

Physics Quantities Having No units

Following physical quantities have no units
Strain, dielectric constant, refractive index, specific gravity, magnification, relative permeability etc.

Note

Physical quantities which are to be added or subtracted must have same units.

Physical Quantities Having Same Units

Quantities	Units	Dimensions
Energy, Work and Heat	$\text{Joule} = 1J = kgm^2s^{-2}$	ML^2T^{-2}
Impulse and Momentum	$Ns = kgms^{-1}$	MLT^{-1}
Electric Field intensity and Potential gradient	$\frac{N}{C} = kgms^{-3}A^{-1}$	$MLT^{-3}A^{-1}$
Plank's constant and Angular momentum	$Js = kgms^2s^{-1}$	ML^2T^{-1}
Pressure, Stress, Modulus of elasticity and Energy density	$\frac{N}{m^2} = kgm^{-1}s^{-2}$	$ML^{-1}T^{-2}$
General gas constant and Molar specific heat	$Jmol^{-1}K^{-1} = kgm^2s^{-2}, mol^{-1}k^{-1}$	$ML^2T^{-2}N^{-1}\theta^{-1}$
Surface tension and spring constant	$\frac{N}{m} = kgs^{-2}$	MT^{-2}
Entropy, Heat capacity and Boltzman constant	$\frac{J}{K} = kgm^2s^{-2}k^{-1}$	$ML^2T^{-2}\theta^{-1}$
Resistance, Reactance of capacitor, reactance of inductor and impedence	$\frac{V}{A} = Jc^{-1}A^{-1} = kgm^2s^{-3}A^{-2}$	$ML^2T^{-3}A^{-2}$

SOME IMPORTANT DERIVED QUANTITIES AND THEIR UNITS.

QUANTITY	SI UNIT	IN-TERMS OF BASE UNITS
Velocity	ms^{-1}	ms^{-1}
Acceleration	ms^{-2}	ms^{-2}
Area	m^2	m^2
Volume	m^3	m^3
Density	kgm^{-3}	kgm^{-3}
Force	N	$kgms^{-2}$

QUANTITY	SI UNIT	IN-TERMS OF BASE UNITS
Work, Energy	J	kgm^2s^{-2}
Power	W	$Js^{-1} = kgm^2s^{-3}$
Pressure, Stress	Pa	$Nm^{-2} = kgm^{-1}s^{-2}$
Torque	Nm	kgm^2s^{-2}
Modulus of elasticity	Nm^{-2}	$kgm^{-1}s^{-2}$
Angular momentum	$J.s$	kgm^2s^{-1}
Impulse, Momentum	$N.s$	$kgms^{-1}$
Surface tension	Nm^{-1}	kgs^{-2}
Boltzman constant	Jk^{-1}	$kgm^2s^{-2}K^{-1}$
Gravitational constant	Nm^2kg^{-2}	$kg^{-1}m^3s^{-2}$
Charge	C	$A.s$
Electric potential	$V = JC^{-1}$	$kgm^2s^{-3}A^{-1}$
Resistance	$ohm = VA^{-1}$	$kgm^2s^{-3}A^{-2}$
Capacitance	$F = CV^{-1}$	$kg^{-1}m^{-2}s^4A^2$
Inductance	$VsA^{-1} = H$	$kgm^2s^{-2}A^{-2}$
Electric flux	Nm^2C^{-1}	$kgm^3s^{-3}A^{-1}$
Magnetic flux	$Wb = NmA^{-1} = T.m^2$	$kgm^2s^{-2}A^{-1}$
Magnetic induction	$T = Wbm^{-2} = Nm^{-1}A^{-1}$	$kgs^{-2}A^{-1}$
Permittivity	$C^2N^{-1}m^{-2}$	$kg^{-1}m^{-3}s^4A^2$
Permeability	$Wbm^{-1}A^{-1}$	$kgms^{-2}A^{-2}$
Entropy	JK^{-1}	$kgm^2s^{-2}K^{-1}$
General gas constant	$Jmol^{-1}K^{-1}$	$kgm^2s^{-2}mol^{-1}K^{-1}$
Electric field intensity	NC^{-1}	$kgms^{-3}A^{-1}$
Plank's constant	$J.s$	kgm^2s^{-1}

SOME IMPORTANT PHYSICAL QUANTITIES

Quantities	Symbol	Value
Time period of geostationary satellite	T	24 hour
Minimum height of geostationary satellite	h	36000 km
Radius of geostationary satellite	γ	42400 km
Speed of sound ($0°C$)	v	$332\ ms^{-1}$
Avogadro's number	N_o	6.02×10^{23} particles
Boltzman constant	K	$1.38 \times 10^{-23}\ Jk^{-1}$
General gas constant	R	$8.314\ Jkg^{-1}mol^{-1}$
Specific heat of water	C	$4186\ Jkg^{-1}k^{-1}$
Permittivity of free space	ϵ_o	$8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$
Charge on electron	$-e$	$-1.6 \times 10^{-19}C$
Charge of proton	$+e$	$+1.6 \times 10^{-19}C$
Charge on alpha particle	$+2e$	$3.2 \times 10^{-19}C$
Columb constant for free space	K	$9 \times 10^9\ Nm^2C^{-2}$
Permeability of free space	μ_o	$4\pi \times 10^{-7}\ Wbm^{-1}A^{-1}$
Potential barrier for Si	V	$0.7V$
Potential barrier for Ge	V	$0.3V$
Speed of light in free space	c	$3 \times 10^8\ ms^{-1}$
Rest mass of photon	m_o	Zero
Charge of photon	q	Zero
Plank constant	h	$6.63 \times 10^{-34}\ Js$
Compton's wavelength	λ_c	$2.43 \times 10^{-12}\ m^{-1}$
Radius of 1st orbit of hydrogen atom	r_1	$0.053\ nm$
Velocity of electron in 1st orbit of hydrogen	V_1	$2.19 \times 10^6\ ms^{-1}$
Ground state energy of hydrogen	E_o	$-13.6\ eV$
Angular momentum of first orbit of hydrogen	L_1	$1.05 \times 10^{-34}\ Js$
Unified atomic mass unit	1u	$1.6606 \times 10^{-27}\ kg$
Mass of electron	m_e	$9.1 \times 10^{-31}\ kg = 0.000554$
Mass of proton	m_p	$1.673 \times 10^{-27}\ kg = 1.007274$
Mass of neutron	m_n	$1.675 \times 10^{-27}\ kg = 1.0086650$
e/m of electron		$1.75 \times 10^{11}\ Ckg^{-1}$
e/m of proton		$9.6 \times 10^7\ Ckg^{-1}$
e/m of neutron		Zero
m/e of neutron		Infinite

UNIT 01 ›› FORCE AND MOTION

Mechanics:

Study of motion of objects is called mechanics. It is divided into two types.

(i) Kinematics (ii) Dynamics

i. **Kinematics:** Study of motion of objects without referring the force.

ii. **Dynamics:** Study of motion of objects taking into consideration of force (the cause of motion)

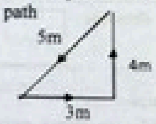
Distance:

Total length of path followed by body is called distance.

- It is scalar quantity. its SI unit is meter.
- It may be zero or positive but never negative.
- Distance of a body in motion can never be zero.

Example: (i) For one vibration distance = 4A
(ii) For one revolution distance = $2\pi r$

Example:
Consider an object moves in a closed path



Total distance = 3+4+5=12m

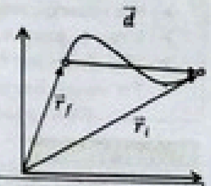
Displacement:

Change in position of a body from its initial position to find position is called displacement.

- It is vector quantity and its direction is from initial to final position.
- Its SI unit is meter

$$\vec{d} = \vec{r}_f - \vec{r}_i$$

(Displacement = Final position vector – Initial position vector)

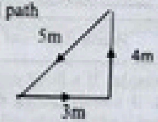


- It may be positive, negative or zero.
- It is shortest distance between initial and final position.
- Displacement is always less than or equal to distance.
- If a body moves in straight line its displacement
- and distance are equal.

Example: (i) For one vibration, displacement = 0
(ii) For one revolution, displacement = 0

[illegible] Initial position [illegible]
[illegible] Displacement [illegible]

Example:
Consider an object moves in a closed path



Total displacement = 0

Example:
Find magnitude of displacement, if a body moves from point P(2, 3) to a point Q(5,7).
(a) 4m (b) 5m (c) 7m (d) 17m

Solution:
$\vec{d} = \vec{r}_f - \vec{r}_i = (5\hat{i} + 7\hat{j}) - (2\hat{i} + 3\hat{j})$
$= 3\hat{i} + 4\hat{j}$
$|\vec{d}| = \sqrt{(3)^2 + (4)^2} = 5m$

Speed:

Distance covered by a body in one second is called speed.

- It is scalar quantity and its SI unit is ms^{-1}.
- It is always positive.
- Speed of a moving object cannot be zero or negative.
- The ratio of speed to magnitude of velocity of an object is always greater than one.

$$Speed = \frac{total\ distance}{time}$$

Motion	Speed
Speed of electro magnetic waves	$3 \times 10^8\ ms^{-1}$
Speed of Earth around the sun	$2.96 \times 10^4\ ms^{-1}$
Speed of moon around the earth	$1 \times 10^3\ ms^{-1}$
Speed of sound in air $(0°C)$	$332\ ms^{-1}$
Speed of running cheetah	$29\ ms^{-1}$

Example:
Consider an object moves in a closed path of radius 2m with time period 3.14 sec. then its speed is
(a) $2ms^{-1}$ (b) $4ms^{-1}$ (c) $4\pi ms^{-1}$ (d) zero

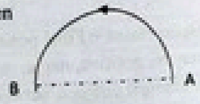
Solution:
Speed $= \frac{Total\ distance}{Time} = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 2}{3.14} = 4ms^{-1}$

Velocity:

Rate of change of displacement of the body is called velocity.

- It is vector quantity. Its SI unit is ms^{-1}.
- It may be positive, negative and zero.
- $|\vec{v}|$ is always less than or equal to speed.
- For straight line motion in one direction $|\vec{v}| = speed$

Example:
Consider an object move from one end to other end of diameter of a circle of radius 1m in 5sec then



$$\vec{v}_{av} = \frac{Total\ displacement}{Total\ time}$$
$$= \frac{2r}{t} = \frac{2 \times 1}{5} = 0.4\ ms^{-1}$$

Average velocity:

$$\vec{v}_{av} = \frac{Total\ displacement}{Total\ time}$$

i. When different displacements and time are given: $v_{av} = \frac{d_1+d_2}{t_1+t_2}$

Example: If a body covers 100m displacement in 4sec and then it covers 200m displacement in 6sec. The average velocity of the body will be:
(a) $10ms^{-1}$ (b) $20ms^{-1}$ (c) $30ms^{-1}$ ✓ (d) $40ms^{-1}$

Solution: $v_{av} = \frac{d_1+d_2}{t_1+t_2} = \frac{100+200}{4+6} = \frac{300}{10} = 30ms^{-1}$

ii. When different velocities and time are given: $v_{av} = \frac{v_1t_1+v_2t_2}{t_1+t_2}$ (put $d = v\,t$)

Example: If a body is moving with velocity $60ms^{-1}$ for 1st hour and $30ms^{-1}$ for next two hours. Then its average velocity of the body will be:
(a) $30kmh^{-1}$ ✓ (b) $60kmh^{-1}$ (c) $90kmh^{-1}$ (d) $120kmh^{-1}$

Solution: $v_{av} = \frac{v_1t_1+v_2t_2}{t_1+t_2} = \frac{(60)(1)+(30)(2)}{1+2} = 30kmh^{-1}$

iii. If velocities are given and time internals are equal $(t_1 = t_2)$: $v_{av} = \frac{v_1+v_2}{2}$

Example: If a car is moving a velocity 20 ms^{-1} for 1st half hour and with velocity $30ms^{-1}$ for next half hour. Then its average velocity of the body will be:
(a) $12.5ms^{-1}$ (b) $25ms^{-1}$ ✓ (c) $50ms^{-1}$ (d) $60ms^{-1}$

Solution: $v_{av} = \frac{v_1+v_2}{2} = \frac{20+30}{2} = \frac{50}{2} = 25ms^{-1}$

iv. When different velocities and displacements are given: $v_{av} = \frac{(d_1+d_2)v_1v_2}{d_1v_2+d_2v_1}$ $\left(t = \frac{d}{v}\right)$

Example: If a velocity of a body is $2ms^{-1}$ for 100m displacement and $4ms^{-1}$ for next 400m displacement. Then average velocity of the body will be:
(a) $3ms^{-1}$ (b) $3.3ms^{-1}$ ✓ (c) $6ms^{-1}$ (d) $6.3ms^{-1}$

Solution: $v_{av} = \frac{(d_1+d_2)v_1v_2}{d_1v_2+d_2v_1} = \frac{(100+400)\times 2\times 4}{(100\times 4)+(400\times 2)} = \frac{4000}{1200} = 3.3ms^{-1}$

v. If displacement are equal $(d_1 = d_2)$. $v_{av} = \frac{2v_1v_2}{v_1+v_2}$

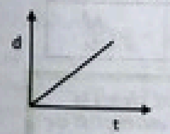
Example: If body covers first half displacement with velocity $10ms^{-1}$ and next half displacement with velocity $20ms^{-1}$. Then its average velocity of the body will be:
(a) $13.3ms^{-1}$ ✓ (b) $15ms^{-1}$ (c) $20.5ms^{-1}$ (d) $0ms^{-1}$

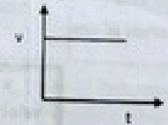
Solution: $v_{av} = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times 10\times 20}{10+20} = \frac{400}{30} = 13.3ms^{-1}$

Uniform Velocity:

If body covers equal displacements in equal intervals of time then it is moving with uniform velocity.

If body is moving with uniform velocity then instantaneous and average velocity are equal.





Instantaneous Velocity:

Velocity of body at any particular instant of time is called instantaneous velocity. OR limiting value of $\frac{\Delta d}{\Delta t}$ as time interval Δt approaches to zero.

Acceleration کا مطلب ہے ہر سیکنڈ میں باڈی کی رفتار کتنی بڑھ رہی ہے یا کم ہو رہی

$$\vec{v}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$$

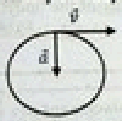
Acceleration:

Rate of change of velocity of a body is called acceleration.

Average Acceleration:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

- It is vector quantity and its SI unit ms^{-2}.
- Its direction is always along the direction of force.
- It may or may not be in the direction of velocity.
- Acceleration is positive and parallel to velocity when velocity of body increases.
- Accelerations is negative and anti-parallel to velocity when velocity of body decreases.
- Acceleration is zero when velocity of the body is constant.
- Acceleration is perpendicular to velocity when speed of the body is constant and only its direction is changing.

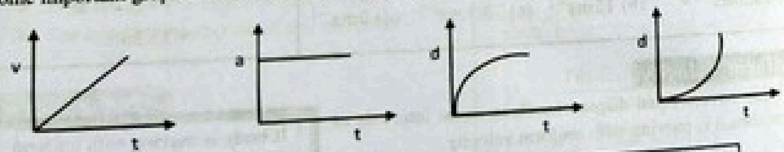


Example: Centripetal acceleration is perpendicular to velocity.

Uniform Acceleration:

Body is moving with uniform acceleration when its velocity changes equally in equal intervals of time.

Some important graph for uniform acceleration are shown in figure below



Instantaneous Acceleration:

Acceleration of a body at any particular instant of time.

$$\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

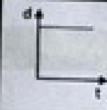
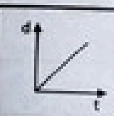
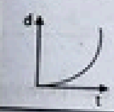
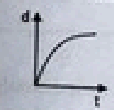
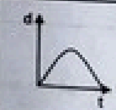
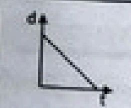
Acceleration is not zero when either magnitude or direction of velocity changes

Acceleration, change in velocity, force and impulse are always in same direction

DISPLACEMENT-TIME GRAPH:

- Graph which illustrates the variation in displacement with time.
- Its slope represents the velocity.

Some Important Examples of Displacement- Time Graph

Graph	Velocity	Slope	Acceleration
	Zero	Zero	Zero
	Constant	Constant	Zero
	Increasing	Increasing	Positive
	Decreasing	Decreasing	Negative
	First decreasing then increases	First decreasing then increases	Negative and constant
	Negative and constant	Negative and constant	Zero

Example: If the displacement time graph of two moving bodies are making angle 30° and 60° with time axis then the ratio between their velocities will be

(a) $1:\sqrt{3}$ (b) 3:1 (c) 1:3 (d) $\sqrt{3}:1$

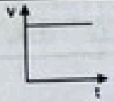
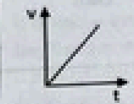
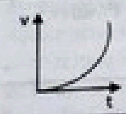
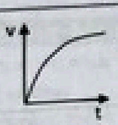
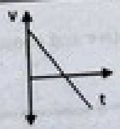
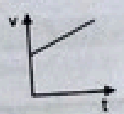
Solution: $\frac{v_1}{v_2} = \frac{\tan 30}{\tan 60}$,

$= \frac{1/\sqrt{3}}{\sqrt{3}} = 1:3$

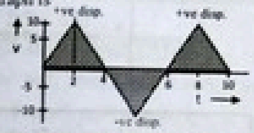
VELOCITY-TIME GRAPH

- Graph which illustrates the variation in velocity of the variation in velocity of the body with time.
- Slope of $v - t$ graph = acceleration.
- Area under $v - t$ graph = distance covered.

Some Important Examples of Velocity- Time Graph

Graph	Velocity	Slope	Acceleration
	Constant ($v_i = v$)	Zero	Zero
	Increasing	Positive and constant	Positive and constant
	Increasing ($v_i = 0$)	Positive and increasing	Positive and increasing
	Increasing ($v_i = 0$)	Positive and decreasing	Positive and decreasing
	First decreasing then increases in opposite direction ($v_i \neq 0$)	Negative and constant	Negative and constant
	Increasing ($v_i \neq 0$)	Positive and constant	Positive and constant

Example 1:
The total distance and displacement covered from the following v-t graph is



(a) 50m, 50m (b) 50m, 30m
(c) 30m, 50m (d) 50m, zero

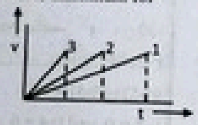
Solution:
distance = total area of triangles

$$= \frac{1}{2}(4)(10) + \frac{1}{2}(2)(10) + \frac{1}{2}(4)(10) = 50$$

(Distance is always positive)

$$displacement = \frac{1}{2}(4)(10) - \frac{1}{2}(2)(10) + \frac{1}{2}(4)(10) = 30$$

Example 2:
If v-t graph for a body is shown in the figure below displacement is maximum for



(a) 1 (b) 2
(c) 3 (d) same for all

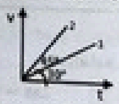
Solution:
$S_1 > S_2 > S_3$

(جس گراف کے نیچے Area زیادہ ہوگا اس کی Displacement زیادہ ہوگا)

$a_3 > a_2 > a_1$

(جس گراف کی slope زیادہ ہوگی اس میں acceleration زیادہ ہوگا)

Example 3:
If v-t graph for two bodies is shown in the figure below then the ratio between their accelerations will be



(a) 1:1 (b) 1: $\sqrt{3}$ (c) 3:1 (d) $\sqrt{3}$: 1

Solution:

$$\frac{a_1}{a_2} = \frac{Tan\ 30^o}{Tan\ 45^o} = \frac{\left(\frac{1}{\sqrt{3}}\right)}{(1)} = \frac{1}{\sqrt{3}}$$

Equations of Motion

1st Equation:

$$v_f = v_i + at$$

(جب distance نہ دیا گیا ہو وقت کی quantity کو معلوم کرنے کے لیے 1st Equation استعمال کریں)

Example:

If a car accelerates uniformly from rest to a velocity $10ms^{-1}$ in 50 sec. Then the acceleration of the car is

(a) $5\ ms^{-1}$ (b) $0.2\ ms^{-2}$
(c) $0.2\ ms^{-1}$ (d) $-0.2ms^{-2}$

Solution:

$v_f = v_i + at$

$a = \frac{v_f - v_i}{t} = \frac{10-0}{50} = 0.2ms^{-2}$

2nd Equation:

$$S = v_i t + \frac{1}{2}at^2$$

(ب) "**final velocity**" نہ دی گئی ہو تو ایسی quantity کو معلوم کرنے کے لیے 2^{nd} Equation استعمال کریں)

Example:

A ball is thrown vertically upward from a 100m high tower with velocity $5ms^{-1}$. The time taken by the ball to reach the ground will be

(a) 4s (b) 5s
(c) 8s (d) 10s

Solution: $S = v_i t + \frac{1}{2}at^2$

$-100 = 5t + \frac{1}{2}(-10)t^2$

$-20 = t - t^2 \rightarrow t^2 - t - 20 = 0$

$t^2 - 5t + 4t - 20 = 0$

$t(t-5) + 4(t-5) = 0$

$t - 5 = 0$ and $t + 4 = 0$

$t = 5sec$

اگر کوئی چیز initial velocity کے opposite ہو تو منفی کے ساتھ لکھیں گے

3rd Equation:

$$2as = v_f^2 - v_i^2$$

(ب) "**time**" نہ دیا گیا ہو تو ایسی quantity کو معلوم کرنے کے لیے 3^{rd} Equation استعمال کریں)

Example:

If a car moving with velocity $10ms^{-1}$ is brought to rest in 25m distance then acceleration of the car will be

(a) $5\ ms^{-2}$ (b) $0.5\ ms^{-2}$
(c) $0.2\ ms^{-2}$ (d) $-0.5\ ms^{-2}$

Solution:

$2as = v_f^2 - v_i^2$

$2a \times 25 = (0)^2 - (10)^2$

$a = -0.5\ ms^{-2}$

- **Note:-These equations are only applicable for linear motion with uniform acceleration.**
- **Note:-Quantities opposite to initial velocity are taken negative.** For example when body is thrown upward then acceleration due to gravity is taken negative.

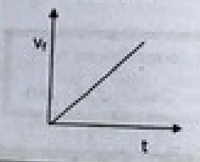
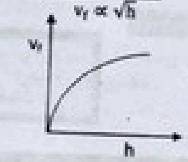
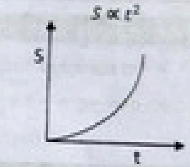
MOTION UNDER THE ACTION OF GRAVITY

(i) When body falls downward:

- When body starts from rest or falls freely or dropped or is released then $v_i = 0$.
- When body is thrown $v_i \neq 0$.
- $a = g = 9.8\ m/s^2 \approx 10m/s^2$

$g = 10\ ms^{-2}$

(مطلب ہر سیکنڈ میں ولاسٹی کی ویلیو 10 m/s بڑھے گی)

Relation between final velocity and time	Relation between final velocity and height	Relation between height and time
$v_f = v_i + gt$	$v_f^2 - v_i^2 = 2gS$	$S = v_i + \frac{1}{2}g t^2$
$v_f = gt$ $(v_i = 0)$	$v_f = \sqrt{2gS}$ $(v_i = 0)$ $v_f = \sqrt{2gh}$ $(S = h)$	$S = \frac{1}{2}gt^2$ $(v_i = 0)$
$v_f \propto t$	$v_f \propto \sqrt{h}$	$S \propto t^2$
		

- Time to reach the ground: $t = \sqrt{\frac{2s}{g}}$

$$t = \sqrt{\frac{2h}{g}} \quad \text{OR} \quad t \propto \sqrt{h}$$

Example:

If a ball falling freely from a height "h" reaches the ground in 10 seconds then the time taken by the ball falling freely from height "2h" will be

(a) 5 sec (b) 7 sec
(c) 14 sec (d) 20sec

Solution:

since $t \propto \sqrt{h}$

when height becomes two time then time will become $\sqrt{2}$ times

$t = \sqrt{2} \times 10 = 14\ sec$

Distance covered in time t:

$$S = \frac{1}{2}gt^2$$

OR $S = 5t^2$

Example

The distance covered by a freely falling body in three second is

(a) 5m (b) 15m
(c) 20m (d) 45m

Solution: $S = 5t^2 = 5 \times 9 = 45m$

Distance covered in nth second:

$$S_{nth} = 10n - 5$$

Example

The distance covered by a freely falling body in 3rd second is

(a) 5m (b) 15m
(c) 20m (d) 45m

Solution: $S = 10n - 5$
$= 10 \times 3 - 5 = 20m$

Note:
If air friction is ignored then motion is independent of mass of the body. For example if two bodies of mass 2kg and 4 kg are dropped from same height they will reach the ground in same time, with same velocity and same acceleration.

When body is thrown upward:

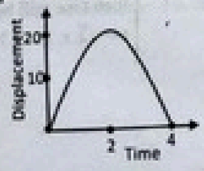
- At maximum height $v_f = 0$.
- $a = -g = -9.8\frac{m}{s^2}$
  $= -10m/s^2$

$$g = -10\ ms^{-2}$$

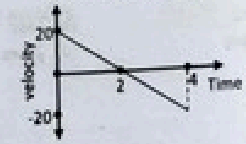
[illegible] $10\ ms^{-1}$ [illegible]

Time to reach maximum height:	Time of Flight	Maximum height attained
$v_f = v_i + at$ At max. height $v_f = 0$ $t = \frac{v_i}{g}$	$S = v_it - \frac{1}{2}gt^2$ For time of flight S = 0 $t = \frac{2v_i}{g}$	$v_f^2 - v_i^2 = -2gh$ At max. height $v_f = 0$ $h = \frac{v_i^2}{2g}$

Displacement Time Graph



Velocity Time Graph



NEWTON'S LAWS OF MOTION

- Sir Isaac Newton published his three laws of motion in 1687 in his book 'principia'.
- Newton's laws are empirical laws.
- Newton's laws are not applicable for sub-atomic particles and motion with speed approaching to speed of light.

Nobody moves or comes to rest itself

(Abu-Ali- Sena)

Ist Law of Motion:

A body at rest will remain at rest and a body moving with uniform velocity, continue its motion unless acted upon by some unbalanced external force.

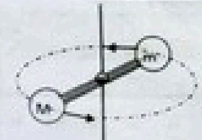
- It gives the qualitative definition of force.
- It is also known as law of inertia.

Inertia:

- ***Property of body tending to maintain its state of rest or uniform motion is called inertia.***
- Mass of a body is quantitative measure of its inertia.

Example:

- A person standing in a bus falls backward due to inertia when bus suddenly starts moving.
- A person standing in a moving bus move forward due to inertia when brakes of the bus is applied suddenly.



A measurement of mass independent of gravity. When two masses are equal the rod will rotate without wobble about its center.

Inertial Frame of reference:

A frame of reference in which Newton's 1st law of motion valid.

- Non- accelerated frame ($\vec{v}$ = constant) is inertial frame.

Example: A car moving with uniform velocity.

If both magnitude and direction of velocity are not changing then frame of reference is 'Inertial'

In thrill machine rides at amusement parks there may be an acceleration of 3g or more. Without head rest this large acceleration may cause serious neck injury due to **inertial effects.**

Non-inertial Frame of reference:

A frame of reference in which Newton's 1st law of motion not valid.

- Accelerated frame ($\vec{v} \neq$ constant) is non-inertial frame.

'If either magnitude and direction of velocity are changing then frame of reference is 'Non-Inertial'

2nd Law of Motion:

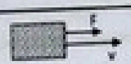
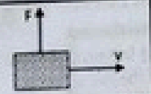
A force applied on a body produces an acceleration in its own direction. This acceleration is directly proportional to applied force and inversely proportional to mass of the body.

$$\vec{a} \propto \frac{\vec{F}}{m} \Rightarrow \boxed{\vec{F} = m\vec{a}}$$

- ➢ Gives the definition of mass.
- ➢ Direction of acceleration is always along the direction of force .

> - No force is needed to continue the motion of the object
> - Force is only needed to change the motion (velocity) of the object.

- **Velocity Change**

Force is parallel to velocity		• $\lvert\vec{V}\rvert$or speed increases. • Direction of velocity remains same.
Force is anti-parallel to velocity		• $\lvert\vec{V}\rvert$or speed decreases. • Direction of velocity remains same.
Force is perpendicular to velocity		• $\lvert\vec{V}\rvert$or speed remains constant. • Only direction of velocity change.
Force acts at some angle with velocity		Both magnitude and direction of velocity changes.

3rd Law of Motion:

"Every action has equal and opposite reaction". $(\vec{F} = -\vec{F})$

> Throwing a package from a boat causes the boat to move backward (Newton's third law)

- ➢ Action and reaction always act on different bodies.
- ➢ According to 3^{rd} law forces always exist in pairs.
- ➢ Action and reaction act on the line joining the two bodies.
- ➢ Action and reaction never balance each other.

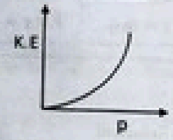
Example: Swimming is an example of Newton's ...... law of motion
(a) first (b) second (c) third ✓ (d) all of these

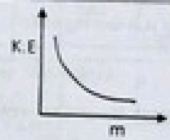
MOMENTUM

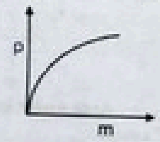
- ➢ The moving object possess a quality by virtue of which it exerts a force on anything that tries to stop it. Quality was called quantity of motion of the body by Newton. Now it is called linear momentum.
- ➢ ***Product of mass of body and its velocity is called momentum.*** $\boxed{\vec{p} = m\vec{v}}$

- ➢ It is vector quantity and its direction is along the direction of velocity.
- ➢ Its SI unit $kgms^{-1}$ or N.s
  - If two bodies have same mass then $p \propto v$. The body moving faster will have larger momentum and vice versa.
  - If two bodies are moving with same speed then $p \propto m$ and heavier body will have larger momentum and vice versa.
  - If two bodies are moving with same momentum then $v \propto \frac{1}{m}$ and lighter body is moving faster and vice versa.
- ➢ K.E and momentum are related as $\boxed{K.E = \frac{p^2}{2m}}$ or $\boxed{p = \sqrt{2m(K.E)}}$

For same/constant mass	For same/constant momentum	For same/constant K.E
$K.E \propto p^2$	$K.E \propto \frac{1}{m}$	$p \propto \sqrt{m}$







Newton's 2nd Law in terms of momentum

" Time rate of change of momentum of a body" is equal to applied force.

$$\boxed{\vec{F} = \frac{\Delta p}{\Delta t}} \quad \text{or} \quad \boxed{\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{t}}$$

- ➢ This is the most general form of Newton's 2^{nd} law of motion.
- ➢ Force is directly proportional to change in momentum of the body.
- ➢ Force is inversely proportional to time of impact.

Examples:

- ➢ Helmet increases time of impact and reduces the force on the head to avoid serious injuring.
- ➢ Hair act like crumple zone, they increase the time of impact thus reduces the force. A force of 5N might be enough to fracture your naked skull (cranium) but with a covering of skin and hair a force of 50N would be needed.
- ➢ Air bags in cars increases the time of impact and thus reduces the force.

> In knocking a bear down Lead bullet is more effective than rubber bullet of same momentum because time of impact for lead bullet is small and it will exert greater force

Impulse:

- Product of force and small interval of time (t) is called impulse.
- $Impulse = \vec{F} \times t$
- It is vector quantity and its direction is along the direction of force.
- Its SI unit is $kgms^{-1}$ or N.s.
- Impulse and momentum have same units.
- Impulse = change in momentum of the body:

$$\vec{F} \times t = m\vec{v}_f - m\vec{v}_i$$

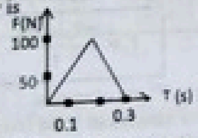
Moving bodies may or may not have impulse
(i). If body is moving with uniform velocity it has no impulse.
(ii). If body is moving with variable velocity its momentum changes and it will have impulse.

If body accelerates from rest	If body brought to rest
$v_i = 0$ $\vec{F} \times t = m\vec{v}_f$	$V_f = 0$ $\vec{F} \times t = -m\vec{v}_i$

Example:
The force required to stop a body moving with a velocity v in a time t is 'F'. The force required to stop the same body in same time if it is moving with velocity 2v is
(a) F (b) 2F✓ (c) 4F (d) F/2

- **Area under force-time graph is equal to impulse.**

Example:
The graph between force and time for which force acts on the body is shown in the figure below. The change in the momentum of the body is



(a) 15 N- sec ✓ (b) 150 N- sec (c) 300 N-sec (d) zero

Solution:
Since area under F-t graph is equal to impulse or change in momentum

$$\Delta p = \frac{1}{2} base \times height$$

$$\Delta p = \frac{1}{2} \times 0.3 \times 100$$

$$\Delta p = 15\ N.sec$$

LAW OF CONSERVATION OF MOMENTUM

- If no external force acts on the system then system is called isolated system.

Total linear momentum of an isolated system remains constant.

If $\vec{F}_{ext} = 0$ then $\frac{\Delta\vec{p}}{\Delta t} = 0$ or $\vec{p} = constant$

(Both magnitude and direction of linear momentum are conserved)

- If masses m_1 and m_2 are moving with velocities $\vec{v}_1$ and $\vec{v}_2$ before the collision and with velocities $\vec{v}_1'$ and $\vec{v}_2'$ after the collision then

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

- When a fighter plane chasing another opens fire is speed and momentum decreases
- When pursued plane returns the fire its speed and momentum increases

Special Case:

If total initial momentum is zero. Then total final momentum is also zero.

Example 1: Spring is compressed between two masses $p_i = 0$, when spring is released then

(i) $\vec{p}_1 = -\vec{p}_2$
(Both masses will have equal and opposite momentum)
(ii) $p = mv \Rightarrow v \propto \frac{1}{m}$
(Lighter body will move faster)
(iii) $K.E = \frac{p^2}{2m} \Rightarrow K.E \propto \frac{1}{m}$
(Lighter body will have greater K.E)

Example 2: Consider a bomb of mass 4kg initially at rest explodes into two pieces of masses 1kg and 3kg as shown in the figure below

(i) $\vec{p}_1 = -\vec{p}_2$ (Both masses will have equal and opposite momentum)

Example: what is ratio between the magnitude of their momentum
(a)1: -1 (b)1: 3 (c) 3: 1 (d) 1: 1✓

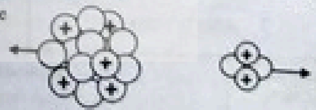
(ii) $p = mv \Rightarrow v \propto \frac{1}{m}$
(Lighter body will move faster)

Example: what is ratio between the magnitude of their velocities
(a)1: -1 (b)1: 3 (c) 3: 1✓ (d) 1: 1

(iii) $K.E = \frac{p^2}{2m} \Rightarrow K.E \propto \frac{1}{m}$
(Lighter body will have greater K.E)

Example: what is ratio between their kinetic energies?
(a)1: -1 (b)1: 3 (c) 3: 1✓ (d) 1: 1

Example 3: Consider a nucleus at rest undergoes an alpha decay as shown in the figure



(i) $\vec{p}_1 = -\vec{p}_2$
(Both nucleus and alpha particle will have equal and opposite momentum)
(ii) $p = mv \Rightarrow v \propto \frac{1}{m}$
(alpha particle will move faster due to smaller mass)
(i). $K.E = \frac{p^2}{2m} \Rightarrow K.E \propto \frac{1}{m}$
(alpha particle will have greater K.E due to smaller mass)

If two bodies collide with eachother and after the collision they move together then their common velocity after the collision is as

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

Example:
A 2500 kg truck moving with velocity 21ms^{-1} collides with a stationary car of mass 1000 kg. The truck and car move together after the impact with common velocity
(a) 5 ms^{-1} (b) 15 ms^{-1}
(c) 10 ms^{-1} (d) 20 ms^{-1}

Solution:

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$v = \frac{2500 \times 21 + 1000 \times 0}{2500 + 1000}$$

$$v = 15 \text{ ms}^{-1}$$

ELASTIC COLLISION

- A collision in which total K.E of the system is conserved is called elastic collision.
- A bouncing ball will rebound to its original height.
- During elastic collision none of K.E is converted into other forms of energy such as sound energy, heat energy etc.

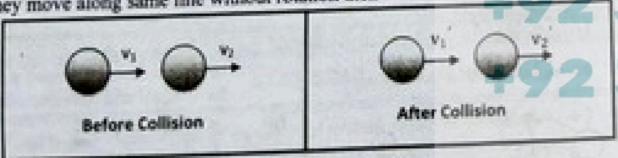
Inelastic Collision:

- A collision in which total K.E of the system is not conserved is called inelastic collision.
- A bouncing ball will not rebound to its original height.
- K.E is converted into other forms of energy.

Note:
Total Energy and momentum remain conserved in both elastic and inelastic collisions.

ELASTIC COLLISION IN ONE DIMENSION

Consider two non-rotating balls moving in same direction collide with each other and after the collision they move along same line without rotation then



$$v_1 + v_1' = v_2 + v_2'$$

(If any three velocities are given you can find the fourth)

- **Relative velocities before and after the collision have same magnitude but opposite direction.**

$$v_1 - v_2 = -(v_1' - v_2')$$

- **Magnitudes of relative velocity of approach is equal to magnitude of relative velocity of separation.**
- Velocities after the collisions are:

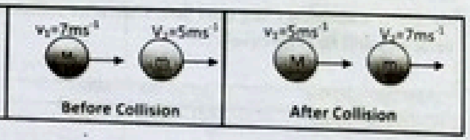
$$v_1' = \frac{(m_1 - m_2)v_1}{m_1 + m_2} + \frac{2m_2v_2}{m_1 + m_2}$$

$$v_2' = \frac{2m_1v_1}{m_1 + m_2} + \frac{(m_2 - m_1)v_2}{m_1 + m_2}$$

Four Special Cases

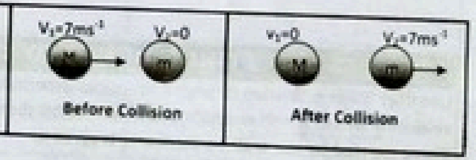
1st Case:

When two bodies of equal mass collide with each other they exchange their velocities after collision.
M = m



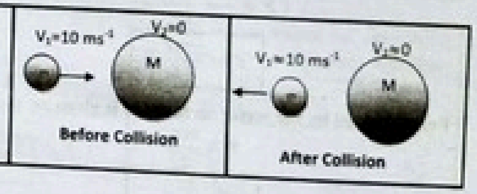
2nd Case:

When a moving ball collides with a stationary ball of equal mass they exchange their velocities after collision.
M = m



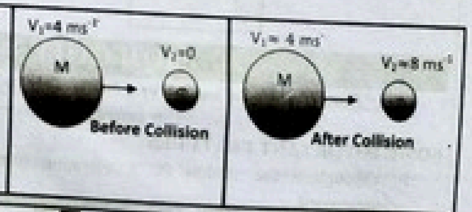
3rd Case:

When a moving lighter ball collides with heavier ball initially at rest then after the collision lighter ball moves backward with same velocity while heavier ball will remain at rest
m ≪ M



4th Case:

When a moving heavier ball collides with lighter ball initially at rest then after the collision velocity of heavier ball remains same while lighter ball will move with double velocity of heavier ball
M ≫ m



Example:
A 70 g ball is moving with a velocity of 9 ms^{-1} collides elastically with another ball of mass 140 g which is initially at rest. Then after collision their velocities will be respectively?

(a) 3 ms^{-1}, -6 ms^{-1} (b) -3 ms^{-1}, 6 ms^{-1}
(c) 6 ms^{-1}, -3ms^{-1} (d) -6 ms^{-1}, 3ms^{-1}

Solution: $V_2 = 0$

$$v_1' = \frac{(m_1 - m_2)v_1}{m_1 + m_2}$$

$$v_1' = \frac{(70-140)9}{70+140} = -3\ ms^{-1}$$

$$v_2' = \frac{2m_1 v_1}{m_1 + m_2}$$

$$v_2' = \frac{2\times70\times9}{70+140} = 6ms^{-1}$$

Example:
A 100 g golf ball is moving with velocity of 20 ms^{-1} collides elastically a stationary 8kg steel ball. Then after collision their velocities will be respectively?

(a) 0 ms^{-1}, 20 ms^{-1} (b) 10 ms^{-1}, 10ms^{-1}
(c) -20ms^{-1}, 0ms^{-1} (d) -10ms^{-1}, 10ms^{-1}

Solution: $V_2 = 0$

$$v_1' = \frac{(m_1 - m_2)v_1}{m_1 + m_2}$$

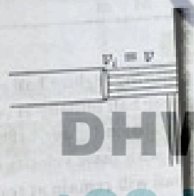
$$v_1' = \frac{(100-8000)20}{100+8000} = -19.5\ ms^{-1}$$

$$v_2' = \frac{2m_1 v_1}{m_1 + m_2}$$

$$v_2' = \frac{2\times100\times20}{100+8000} = 0.5ms^{-1}$$

FORCE DUE TO WATER FLOW

Consider water is flowing through a horizontal pipe with velocity v strikes a wall normally and comes to rest then



Force exerted by the wall on the water is given as

$$F = \frac{mv_f - mv_i}{t}$$

$$F = \frac{-mv}{t}$$

Force exerted by the water on the wall is given as

$$F = \frac{mv_i}{t}$$

$$F = \frac{m}{t}v$$

PROJECTILE MOTION

"Two dimensional motion of a body under constant acceleration due to gravity is called projectile motion".

SOME IMPORTANT FEATURES:

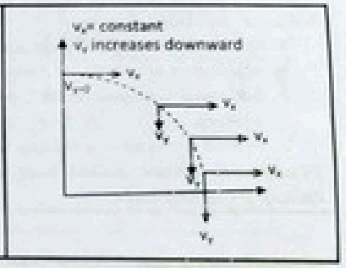
- Throughout the motion the acceleration is constant ($a = g$) and directed vertically downward.

- Vertical components of velocity continuously changes with time.
- Horizontal component of the velocity always remains constant.
- Horizontal component of acceleration remains zero.
- For ***short range*** the trajectory or path of projectile is **parabola.**
- For ***long range*** the trajectory or path of projectile is **elliptical.**

Case-1:
If a body is projected in horizontal direction with velocity v_x.

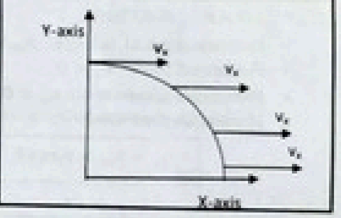
- It moves forward due to inertia
- It moves downward due to gravity

Example: A bomber moving in horizontal direction drops a bomb when it is just above its target but he misses the target due to
(a) horizontal velocity (b) inertia
(c) gravity (d) both a and b✓



Horizontal Motion:

- If air friction is ignored then force along X-axis remains zero : $F_x = 0$
- Horizontal component of acceleration is always zero. $a_x = 0$
- Horizontal component of velocity remains constant (horizontal component of velocity is same at all the points)

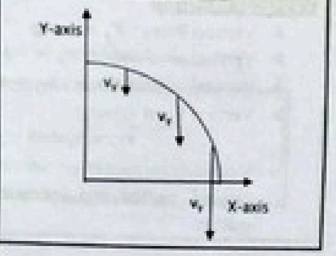


Vertical Motion:

- Force along y-axis: $F_y = mg$
- Acceleration along y-axis: $a_y = g$
- Initial vertical velocity is zero and it increases with time.
- Vertical final velocity: $v_{fy} = gt$
 OR
 $v_{fy} = \sqrt{2gh}$
- Vertical distance: $h = \frac{1}{2}gt^2$

Initially y-component of velocity is zero then increases linearly with time

$$v_{fy} = gt$$



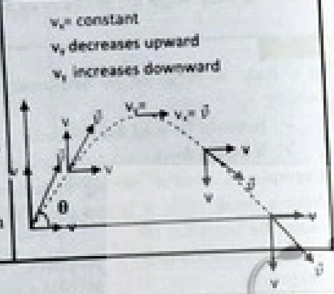
- Resultant acceleration: $a = \sqrt{a_x^2 + a_y^2} = \sqrt{0 + g^2} = g$
- Resultant velocity: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_i^2 + g^2t^2} = \sqrt{v_i^2 + 2gh}$

Case-2:

Consider Body is projected at an angle "θ" with the horizontal and initial velocity v_i

- θ is known as angle of projection
- v_i is known as velocity of projection
- horizontal component of velocity remains constant
- vertical component of velocity vary with time

Example: A ballistic missile fired from a certain distance at a certain angle.



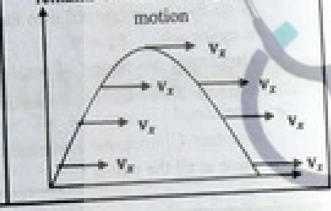
Horizontal motion:

If air friction is ignored then.

- Horizontal initial velocity: $v_{ix} = v_i\cos\theta$
- Horizontal force: $F_x = 0$
- Horizontal acceleration: $a_x = 0$
- Horizontal final velocity:

$$v_{fx} = v_{ix} = v_i\cos\theta$$

Horizontal component of velocity remains constant throughout the motion



- Resultant acceleration: $a = \sqrt{a_x^2 + a_y^2} = \sqrt{0 + (-g)^2} = g$

Resultant velocity: $v = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(v_i\cos\theta)^2 + (v_i\sin\theta - gt)^2}$

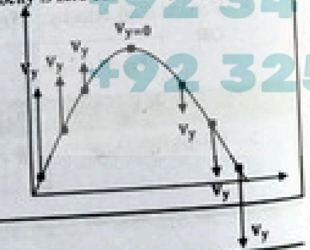
Vertical motion:

- Vertical Force: $F_y = -mg$
- Vertical acceleration: $a_y = -g$
- Vertical initial velocity: $v_{iy} = v_i\sin\theta$
- Vertical Final velocity:

$$v_{fy} = v_i\sin\theta - gt$$

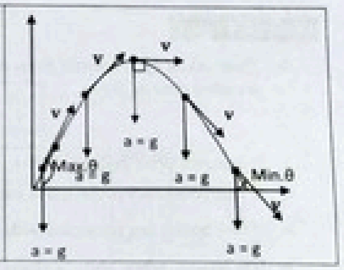
- Vertical component of velocity decreases in upward motion and increases in downward motion

At maximum height the y-component of velocity is zero but velocity is not zero



Angle between velocity & acceleration:

- Angle between velocity and acceleration of projectile is maximum at point of projection.
- Angle between velocity and acceleration decreases with time.
- At maximum height velocity and acceleration are perpendicular to each other.
- At the point where the body hits the ground the angle between velocity and acceleration is minimum.



At max height

- Y-component of velocity = 0.
- Velocity is minimum and $v = v_i\cos\theta$.
- Angle between velocity and acceleration is 90°.

$$K.E = \frac{1}{2}m(v_i\cos)^2 = \frac{1}{2}mv_i^2\cos^2\theta$$

$$P.E = mgh = mg\left(\frac{v_i^2\sin^2\theta}{2g}\right) = \frac{1}{2}mv_i^2\sin^2\theta$$

Maximum Height:

(Maximum distance covered by a projectile in vertical direction is called maximum height)

$$H = \frac{v_i^2\sin^2\theta}{2g}$$

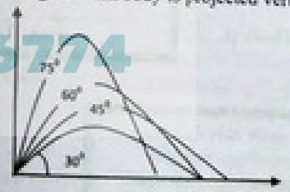
- It depends upon initial velocity, angle of projection and acceleration due to gravity.
- If two bodies are projected at same angle then

$$H \propto v_i^2 \quad and \quad \left(\frac{H_1}{H_2} = \frac{v_1^2}{v_2^2}\right)$$

- If two bodies are projected with same speed then

$$H \propto \sin^2\theta \quad and \quad \left(\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2}\right)$$

- H is largest when body is projected vertically upward ($\theta = 90°$) and $H_{max} = \frac{v_i^2}{2g}$



یاد رکھیں

Angle کا 90° کے قریب ہو گا Height اتنی زیادہ ہو گی۔

Time of Flight:

➢ ***Time taken by projectile from its point of projection to the point where it hit the ground is called time of flight.***

$$t = \frac{2v_i \sin\theta}{g}$$

➢ It depends upon initial velocity, angle of projection and acceleration due to gravity.

➢ If two bodies are projected at same angle then $t \propto v_i \left(\frac{t_1}{t_2} = \frac{v_1}{v_2}\right)$.

➢ If two bodies are projected with same speed then $t \propto \sin\theta \left(\frac{t_1}{t_2} = \frac{\sin\theta_1}{\sin\theta_2}\right)$.

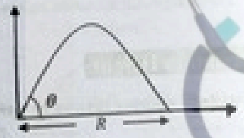
➢ It is largest when body is projected vertically upward $(\theta = 90^\circ)$ and $t_{max} = \frac{2v_i}{g}$.

Note
Time to reach maximum height is $\frac{v_i \sin\theta}{g}$.

یاد رکھیں
Angle کا 90° کے قریب ہو تو Body کا ٹائم آف فلائٹ زیادہ ہوتا ہے

Horizontal Range:

Maximum distance covered by projectile in horizontal direction is called range

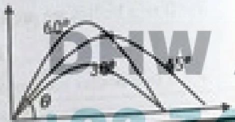


$$R = v_{ix}t \quad \text{or} \quad R = \frac{v_i^2 \sin 2\theta}{g}$$

➢ It depends upon initial velocity, angle of projection and acceleration due to gravity.

➢ If two bodies are projected with same speed then

$$R \propto \sin 2\theta \quad \text{and} \quad \left(\frac{R_1}{R_2} = \frac{\sin 2\theta_1}{\sin 2\theta_2}\right)$$



➢ If bodies are projected at same angle then

$$R \propto v_i^2 \quad \text{and} \quad \left(\frac{R_1}{R_2} = \frac{v_1^2}{v_2^2}\right)$$

یاد رکھیں
Angle کا 45° کے قریب ہو تو range زیادہ ہو گی

➢ Range of projectile is maximum when body is projected at 45° and $R_{max} = \frac{v_i^2}{g}$

Note: For some of initial speed, angle of projections for which $\theta_1 + \theta_2 = 90^\circ$, Horizontal ranges are equal.
Example: At 30° and 60° horizontal ranges are equal.

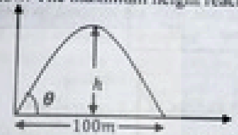
Relation between maximum height and range:

$$H = \frac{1}{4} R \tan\theta$$

➢ $H = \frac{1}{4} R_{max}$ (For maximum range $\theta = 45^\circ$ and $\tan 45^\circ = 1$)

➢ For $\theta = 76^\circ$ maximum height and range are equal.

Example:
The trajectory of a ball thrown at an angle of 45° is shown in the figure below. The maximum height reached by the ball is



(a) 100m (b) 200m
(c) 400m (d) 25m

Solution:
$H = \frac{1}{4} R \tan\theta$
$= \frac{1}{4} \times 100 \times \tan 45$
$H = 25m$

Example:
The range of projectile will be four times the height of projectile at an angle of
(a) 25° (b) 45°✓ (c) 76° (d) 82°

Solution:
$H = \frac{1}{4} R \tan\theta$
$H = \frac{1}{4} \times 4H \tan\theta$
$\tan\theta = 1$ or $\theta = 45^\circ$

Example:
If the range of projectile and height of projectile are related as $R = \frac{4H}{\sqrt{3}}$ then angle of projection will be
(a) 30° (b) 45° (c) 60°✓ (d) 76°

Solution:
$H = \frac{1}{4} R \tan\theta$
$H = \frac{1}{4} \times \frac{4H}{\sqrt{3}} \tan\theta$
$\tan\theta = \sqrt{3}$

Relation between maximum height and time of flight:

$$H = \frac{1}{8} g t^2$$

Example:
If the ratio between time of flight of two different bodies is 2:3 then the ratio between their maximum height will be
(a) 2:3 (b) 4:9✓ (c) $\sqrt{2}:\sqrt{3}$ (d) 1:1

Solution:
$H \propto t^2$
Ratio between height is determined by taking square of the ratio between time

Example:
If time of flight of a projectile is four seconds the maximum height reached by projectile is

(a) 10m (b) 20m ✓ (c) 40m (d) 50m

Solution:

$H = \frac{1}{8}gt^2$

$H = \frac{1}{8} \times 10 \times (4)^2$

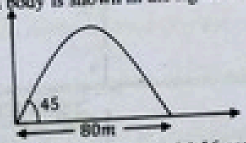
$H = 20m$

Relation between Range and Time of flight:

$R \tan\theta = \frac{1}{2}gt^2$

$R_{max} = \frac{1}{2}gt^2$

Example:
Trajectory of a body is shown in the figure below. What is its time of flight?



(a) 4sec (b) 8sec (c) 16 sec (d) $\sqrt{8}sec$

Solution:

$R \tan\theta = \frac{1}{2}gt^2$

$80 \times Tan45$

$= \frac{1}{2} \times 10 \times t^2$

$t^2 = 16$

Ballistic Flight:

Ballistic flight is that in which projectile is given an initial push and then allowed to move freely due to inertia and under action of gravity.

Ballistic Missile:

The unpowered and unguided missile is called ballistic missile.

- The ballistic missiles are useful only for short ranges.
- Powered and remote controlled guided missiles are used for long ranges and greater precision.

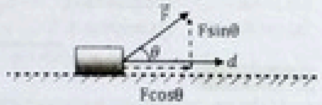
Ballistic Trajectory:

The path followed by ballistic missile is called ballistic trajectory.
If we consider earth as a flat surface, the shape of trajectory is parabolic and its range is short.
If we consider earth spherical, the shape of trajectory is elliptical.

UNIT 02 ⟩⟩ WORK AND ENERGY

WORK DONE BY CONSTANT FORCE

"The product of magnitude of displacement and component of force in the direction of displacement is called work done on the body."



- Work is scalar quantity and its unit is Joule.

$1J = Kgm^2s^{-2}$

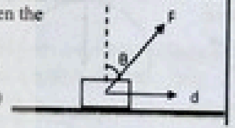
- In cgs system unit of work is erg.

$1erg = 10^{-7}J$

$W = (F\cos\theta)(d)$

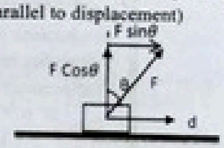
$W = Fd\cos\theta = \vec{F}.\vec{d}$

Example:
If a body is moving under the action of constant force as shown in the figure below then the work done on the body is



(a) Fd Cosθ (b) Fd Sinθ
(c) Fd Tanθ (d) - Fd Cosθ

Solution: W=displacement(component of force parallel to displacement)
= Fd sinθ



Example:
At what angle between force and displacement more work is done.

(a) 20° ✓ (b) 40°
(c) 60° (d) 80°

Solution:
smaller the angle, larger the value of Cosθ
Work will be maximum when angle is smallest

Example:
At what angle between force and displacement the work done on the body is just 50% of maximum work

(a) 30° (b) 45°
(c) 60° (d) 90°

Solution:
Sine value of Cos60°=1/2
Hence work done is 50% when θ = 60°

Positive work :

If the angle between force and displacement is less than 90° then work is positive.

Example:
If the angle between force and displacement of the body is 30° then work done will be

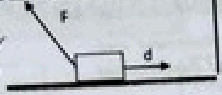
(a)positive ✓ (b) maximum (c) negative (d) minimum

Negative work :

If the angle between force and displacement is greater than 90° then work is negative. (Work done by friction is always negative) .

Example:
If a body is moving under the action of constant force as shown in the figure below then the work done on the body is

(a) positive (b) negative✓
(c) maximum (d) minimum



Solution:
Since the angle between force and displacement is greater than 90°. Hence work done will be negative.

Zero work :

1. **If the angle between force and displacement is 90° then work is zero.**

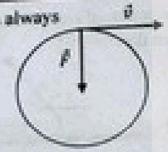
Example:
Work done by magnetic force on a moving charge in a magnetic field is always
(a) positive (b) negative (c) zero✓ (d) none of these

Solution:
Magnetic force on a moving charge is always perpendicular to velocity or displacement.

Work done by centripetal force is always zero because force is always Perpendicular to velocity or displacement of the body.

$W = Fd\cos 90°$

$W = 0$



2. **If the body covers no displacement then work is zero.**

Example:
Work done by a man pushing the rigid wall is zero because
(a) force is zero (b)✓ displacement is zero
(c) force and displacement are parallel (d) force and displacement are perpendicular

3. **If force acting on the body is zero (body is moving with uniform velocity or speed) then work done on it is zero**

Example:
If a car is moving with uniform velocity then work done on the car by Engine ,Friction and net force is respectively
(a) positive , negative, positive
(b) positive , negative, negative
(c) positive , negative, zero✓
(d) positive , zero, positive

Solution:
Work done by $Engine = Fd\cos 0° = +Fd$
Work done by $Friction = Fd\cos 180° = -Fd$
Net work done = 0
(As car is moving with uniform velocity so $a = 0$ and $F_{net} = 0$)

Work done in term of rectangular components

$$W = \vec{F}.\vec{d} = (F_x d_x + F_y d_y + F_z d_z)$$

Example:
If a force $\vec{F} = 4\hat{i} + 6\hat{j}$ displaces the body from point $p(2, 3)$ to a point $Q(5, -2)$ then what is work done.
(a) 9 J (b) -9 J
(c) 18 J (d) -18 J

Solution:

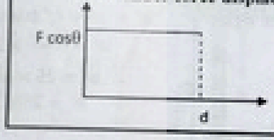
$\vec{d} = \vec{r}_f - \vec{r}_i$

$= (5\hat{i} - 2\hat{j}) - (2\hat{i} + 3\hat{j}) = 3\hat{i} - 5\hat{j}$

$W = \vec{F}.\vec{d} = (4\hat{i} + 6\hat{j}).(3\hat{i} - 5\hat{j})$

$= (4)(3) + (6)(-5) = -18\ J$

Area under force displacement graph is equal to work done.



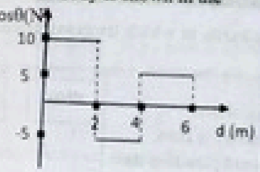
$Area\ of\ rectangle = length \times width$

$= (F\cos\theta)\ (d)$

$= Fd\cos\theta$

$= work\ done$

Example:
The graph between displacement and component of force in the direction of displacement for a body is shown in the figure below.
Work done on the body is
(a) 20J (b) 30J
(c) 40J (d) 50J



Solution:

$W = total\ area$

$W = (10 \times 2) - (2 \times 5) + (2 \times 5)$

$W = 20 - 10 + 10$

$W = 20J$

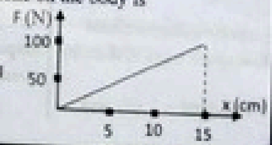
WORK DONE BY VARIABLE FORCE

In most cases force is not constant throughout the displacement but it is variable.

Examples:

(i) **In stretching a spring. Work is done by variable force ($F \propto x$).**

Example: Force – displacement graph for a body is shown in the figure below. Work done on the body is
(a) 7.5J (b) 15J
(c) 750J (d) 1500J



Solution:

$W = area\ of\ triangle$

$W = \frac{1}{2} \times 100 \times 15 \times 10^{-2}$

$W = 7.5\ J$

(ii) A rocket moving away from earth. Work is done by variable force ($F \propto \frac{1}{r^2}$).

(iii) Two positive point charges are brought closer to each other. Work is done by variable force ($F \propto \frac{1}{r^2}$).

To calculate work done by variable force there are two methods.

I. we divide the path into small intervals so that in each interval force approximately remains constant then we calculate work done during each interval by using relation

$$W_i = \vec{F_i}.\vec{\Delta d_i} = F_i \Delta d_i cos\theta_i \text{ then}$$

$$W_t = \sum_{i=1}^{n} F_i \Delta d_i cos\theta_i$$

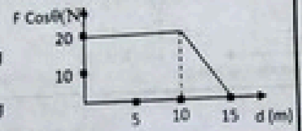
II. We plot a graph between force and displacement and $W_t = Total\ area\ under\ the\ graph.$

Example:

Force – displacement graph for a body is shown in the figure below. Work done on the body is

(a) 50J (b) 200J

(c) 150J (d) 250J



Solution:

$$W = area\ of\ trapezium$$

$$W = \frac{10+15}{2} \times 20$$

$$W = 25 \times 10$$

$$W = 250J$$

WORK DONE BY GRAVITATIONAL FORCE

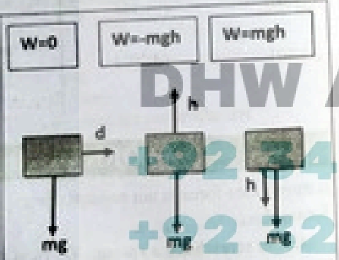
➢ **Region or space around the earth in which its gravitational force acts on the body is called gravitational field.**

In the Gravitational Field:

(i) If a body is displaced in horizontal direction, work done by gravity is zero.

(ii) If body is displaced in upward direction then work done by gravity is negative **($W = -mgh$).**

(iii) If body is displaced in downward direction then work done by gravity is positive **($W = mgh$)**



Example:

A person holding a 10kg bag covers a displacement 5m in horizontal direction. How much work is done by gravity

(a) 50J (b) 500J (c) -500J (d) zero

Solution:

Since force of gravity is perpendicular to displacement

Hence work is zero

(i) Work done by gravity is independent of path followed.

Along path1:

$$W_{AB} = W_{AD} + W_{DB} = 0 + (-mgh) = -mgh$$

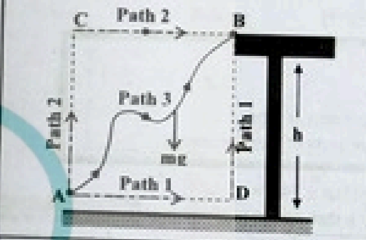
Along path2:

$$W_{AB} = W_{AC} + W_{CB} = (-mgh) + 0 = -mgh$$

Along path3:

$$W_{AB} = -mg(total\ verical\ displacement)$$

$$W_{AB} = -mgh$$

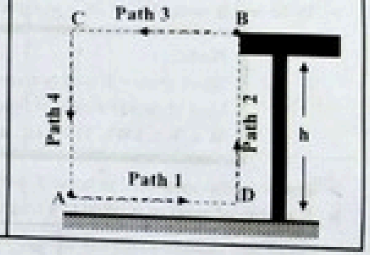


(ii) Work done by Gravity in a closed path is zero.

$$W_{ADBCA} = W_{AD} + W_{DB} + W_{BC} + W_{CA}$$

$$W_{ADBCA} = 0 + (-mgh) + 0 + (+mgh)$$

$$W_{ADBCA} = 0$$



Conservative Force:

➢ The force for which work done in closed path is zero is called conservative force.

➢ The force for which work is independent of the path followed by the body is called conservative force.

Examples:

Gravitational force, electric force, elastic force. are conservative forces.

Non-Conservative Force:

➢ Work done depends upon the path followed (Longer the path, larger the work).

➢ Work done in closed path is not equal to zero.

➢ Work done on aeroplane by air friction is not zero in a closed path and longer the path followed by aero plane, larger the work done by air friction.

Examples:

➢ Friction, Viscous force, Normal force, Tension, Air resistance, Propulsive force of rocket and motor are non-conservative forest.

POWER

"Rate at which work is being done.
OR Work done by a body per unit time is called power of the body."

- Average power: $P_{av} = \frac{\text{Total work}}{\text{Total time}} = \frac{W}{t}$
- **It is a scalar quantity. It is equal to dot product of force and velocity.**

$$P = \vec{F}.\vec{v} = FV\cos\theta$$

- Its SI unit is watt ($1W = Js^{-1} = kgm^2s^{-3}$).

> **Note:**
> Work done = Power × time
> Unit of power × unit of time = unit of work
> W.s, Wh, kWh, MWh etc. are units of work or energy.

- Power is also measured in horse power.(1hp = 746 W)
- If a body of mass m is lifted to a height h then power is given as

$$P = \frac{mgh}{t}.$$

If work done is equal to change in K.E	If work done is equal to change in P.E
$P = \frac{\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}{t}$	$P = \frac{mgh_2 - mgh_1}{t}$

Kilo-Watt Hour:

"One Kilowatt hour is work done in one hour by an agency whose power is one kilowatt."

1Wh = 3.6 kJ	1kWh = 3.6MJ	1MWh = 3.6GJ

Kilowatt hour is the commercial unit of electrical energy.

ENERGY

Capacity of a body to do work is called energy.

- There are many types of energy such as K.E, P.E , sound energy, heat energy, electrical energy, chemical energy, solar energy etc.
- Mechanical energy can either be K.E or P.E
- It is a scalar quantity
- Its SI unit is joule ($1J = N.m = kg\ m^2s^{-2}$)
- Work and energy have same units.

Kinetic Energy

"Energy possessed by a body due to its motion is called kinetic energy."

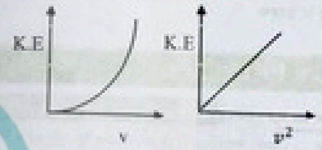
- Kinetic energy cannot be negative.

In terms of velocity

$$K.E = \frac{1}{2}mv^2$$

It depends upon mass of the body and its velocity

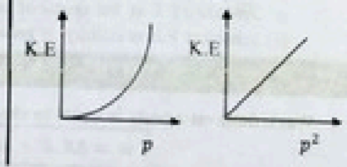
$K.E \propto v^2$
And $K.E \propto m$



In terms of momentum

$$K.E = \frac{P^2}{2m}$$

$K.E \propto p^2$
And $K.E \propto \frac{1}{m}$



Potential Energy

Energy possessed by a body due to its position in the force field or due to its constrained state.

- P.E is always determined relative to some reference point where P.E is taken zero.
- Reference point can be chosen anywhere.

> **NOTE:**
> P.E due to force of attraction is always negative
> P.E due to force of repulsion is always positive

Examples:

- Gravitational $P.E = mgh$ (if surface of earth is taken as a reference point)
- Absolute gravitational $P.E = -\frac{GMm}{r}$ (if reference point is taken at infinity where force of gravity is zero)
- Elastic $P.E = \frac{1}{2}kx^2$
- Electrical P.E between two point charges = $\frac{kq_1q_2}{r}$
- Electrical P.E stored in capacitor = $\frac{1}{2}CV^2$
- Magnetic P.E stored in Inductor = $\frac{1}{2}LI^2$
- ***Energy stored in compressed or stretched spring is called elastic P.E.***

ABSOLUTE POTENTIAL ENERGY

"Absolute P.E at any point is defined as work done by gravitational field in moving the object from that point to infinity where force of gravity becomes zero."

- Absolute P.E at any distance r from the center of earth is given as

$$P.E = -\frac{GMm}{r}$$ *(absolute P.E is always negative)*

- Absolute P.E on the surface of earth is given as

$$P.E = -\frac{GMm}{R}$$

- Absolute P.E at any height h from the surface of earth is given as

$$P.E = -\frac{GMm}{R+h}$$ *(by increasing the height absolute P.E increases)*

- Absolute P.E at the center of earth is **zero**.
- Absolute P.E at infinity is **zero**.

WORK ENERGY PRINCIPLE

Work done on a body is equal to change in its energy.

$$W = \Delta K.E + \Delta P.E$$

- If work done on the body is positive then its energy increases.
- If work done on the body is negative then its energy decreases.
- If work done on the body is zero then its energy remains constant.

Case 1:

If work done by conservative force is zero then P.E=0 and $W = \Delta K.E$

OR $Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

- If work done on the body is positive then its K.E increases.
- If work done on the body is negative then its K.E decreases.
- If work done on the body is zero then its K.E remains constant.

If body is accelerated from rest then $v_i = 0$

$$Fd = \frac{1}{2}mv_f^2$$

If body is brought to rest $v_f = 0$

$$Fd = -\frac{1}{2}mv_i^2$$

Case 2:

If work is done by conservative force while keeping the body in equilibrium then K.E=0 and $W = \Delta P.E$

OR

$$W = mgh_2 - mgh_1$$

- If work done on the body is positive then its P.E increases.
- If work done on the body is negative then its P.E decreases.
- If work done on the body is zero then its P.E remains constant.

Example:

If 1000N force is required to stop a car moving with velocity 10ms⁻¹ then the force required to stop the car in same distance when it is moving with velocity 20ms⁻¹ will be

(a) 500N (b) 1000N
(c) 2000N (d) 4000N

Solution:

$$Fd = -\frac{1}{2}mv_i^2$$

As 'd' and are same so

$F \propto v^2$

If v is doubled then F becomes 4 times ($4\times 1000 = 4000N$)

Example:

What is force required to accelerate an object of mass 1kg from rest to a velocity 4ms⁻¹ in a distance 10m

(a) 0.4 N (b) 0.8 N
(c) 4 N (d) 8 N

Solution:

$$Fd = \frac{1}{2}mv_f^2$$

$$F \times 10 = -\frac{1}{2} \times 1(4)^2$$

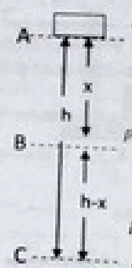
$$F = 0.8N$$

INTERCONVERSION OF K.E AND P.E

In the absence of air friction

If a body of mass m falls under the action of gravity from a height h as shown in the figure.

- Loss in P.E is equal to gain in K.E.
- Total energy always remains constant.



$P.E = mgh, K.E = 0$

$total\ energy = mgh$

$P.E = mg(h-x), K.E = mgx$

$total\ energy = mgh$

$P.E = 0, K.E = mgh$

$total\ energy = mgh$

After falling downward distance X

- Loss in $P.E = mgx$
- % loss in P.E $= \frac{x}{h} \times 100$
- Gain in K.E $= mgx$
- % Gain in K.E $= \frac{x}{h} \times 100$
- Gain in speed/velocity $= \sqrt{2gx}$

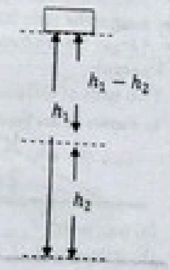
Velocity of object falling under gravity

$Gain\ in\ K.E = loss\ in\ P.E$

$$\frac{1}{2}m(v_2^2 - v_1^2) = mg(h_1 - h_2)$$

- If at height h_1 body is moving with velocity v_1 then at height h_2 velocity of the body will be

$$v_2 = \sqrt{2g(h_1 - h_2) + v_1^2}$$



If body falls from rest then $v_1 = 0$

$$v_2 = \sqrt{2g(h_1 - h_2)}$$

$$v \propto \sqrt{(h_1 - h_2)}$$

$$v \propto \sqrt{vertical\ distance}$$

- Velocity only depends upon initial and final height and is independent of mass of the body and path followed.
- If body reaches the ground then $h_1 = 0$

$$v = \sqrt{2gh} \qquad v \propto \sqrt{h}$$

LAW OF CONSERVATION OF ENERGY
Energy cannot be destroyed. It can be transformed from one kind to another but total amount of energy always remains constant.

Example:
Two balls of different masses rolls down from a frictionless plane as shown in the figure below then which balls is moving faster

(a) bigger ball (c)✓ both are moving with same speed
(b) smaller ball (d) depend upon path length

Solution:
$$v = \sqrt{2gh}$$
Velocity only depends upon height and is independent of mass of the body and path followed

In the Presence of friction

If body falls downward then
- Loss in P.E = Gain in K.E + Work done against the friction.

$$mgh = \frac{1}{2}mv^2 + fd$$

- Gain in speed: $v = \sqrt{2\left(gh - \frac{fd}{m}\right)}$
- Larger the mass, greater the speed.
- Larger the path length, smaller the speed.
- Larger the friction, smaller the speed

If body is thrown upward then
- Loss in K.E = Gain in P.E + Work done against the friction.

$$\frac{1}{2}mv^2 = mgh + fd$$

Example:
Two balls of different masses rolls down from a rough plane as shown in the figure below then which balls is moving faster

(a) bigger ball✓ (c) both are moving with same speed
(b) smaller ball (d) depend upon path length

Solution:
In presence of friction
- Larger the mass, greater the speed.
- smaller the path, larger the speed

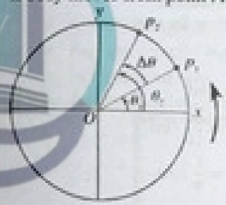
UNIT 03 ›› ROTATIONAL AND CIRCULAR MOTION

Circular Motion:
"Motion of a body moving in circular path or motion of a body whose distance from axis of rotation remains constant is called circular motion".

Examples:
- Motion of satellites around the earth.
- Motion of car moving on a circular track.
- Motion of stone tied with a string, rotating in a circular path.

ANGULAR DISPLACEMENT
"Angle subtended at the center in small internal of time or angle Δθ which gives the change in angular position of a body is called angular displacement".

Examples:
If body moves from point A to B on circular path then its angular displacement is $\Delta\theta$



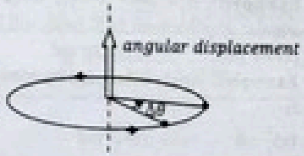
Note One radian is the angle between two radii which cut off on the circumference an arc equal to radius

Note
Rotational motion is either two or three dimensional motion and cannot be one dimensional.

- SI unit of angular displacement is radian and other units are degree, revolution etc.
- For small value, angular displacement is vector quantity.
- For large value angular displacement is not vector because it does not obey the vectors laws such as commutative law $(\theta_1 + \theta_2 = \theta_2 + \theta_1)$.

Example: if a body moves from one end of the diameter to other then angular displacement of the body will be
(a) 90° (b) 180°✓ (c) 270° (d) 360°

Right hand rule :
Direction of angular displacement is along axis of rotation and it is determined by right hand rule. (Rotate fingers in direction of rotation while keeping the thumb erect then thumb indicates the direction of angular displacement).

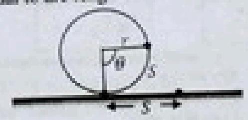


Rotation between Linear and Angular displacement:

If a particle moving in circular path of radius r, covers an arc length S and angular displacement θ then

$$S = r\theta$$

If a body is rolling without slipping then its linear distance is equal to arc length.



یاد رکھیں

$S = r\theta$ صرف تب hold کرے گا جب θ کی ویلیو Radians میں ہوگی۔

Examples:
If a wheel of radius 0.5m is rolling without slipping then its linear distance covered in 3-revolutions will be
(a)1.5m (b)2m
(c) 5.4m (d) 9.4m

solution:
$s = r\theta = 0.5 \times 3 \times 2\pi$
$= 3\pi m = 9.4\ m$
$(1\ rev = 2\pi rad)$

Conversion of degree into radian:

- $30° = 30 \times \frac{\pi}{180} = \frac{\pi}{6}\ rad$
- $45° = 45 \times \frac{\pi}{180} = \frac{\pi}{4}\ rad$
- $60° = 60 \times \frac{\pi}{180} = \frac{\pi}{3}\ rad$

Conversion of Radian into radian:

- $\frac{\pi}{2}$rad $= \frac{180°}{2} = 90°$
- πrad $= 180°$
- $\frac{3\pi}{2}$rad $= \frac{3\times180°}{2} = 270°$

Some important conversions

- $1\ rev = 360° = 2\pi\ rad$
- $1\ rad = 57.3°$
- $1° = 0.0174\ rad$
- $30° = \frac{\pi}{6}\ rad = \frac{1}{12}\ rev$
- $45° = \frac{\pi}{4}\ rad = \frac{1}{8}\ rev$
- $60° = \frac{\pi}{3}\ rad = \frac{1}{6}\ rev$
- $90° = \frac{\pi}{2}\ rad = \frac{1}{4}\ rev$
- $180° = \pi\ rad = \frac{1}{2}\ rev$

Example1: if a body covers three revolutions in 5seconds then its angular displacement in SI units will be
(a) 2π rad (a) 3π rad (a) 6π rad (a) 9π rad

Solution:
3 rev = $3\times 2\pi$ rad
$= 6\pi$ rad

Example2: Angle subtended by minute hand of the clock in 15 minutes is
(a)$\frac{\pi}{2}$ rad (a)$\frac{3\pi}{2}$ rad (a)$\frac{6\pi}{5}$ rad (a) 2π rad

Solution:
Angle in 15min. = 90°
$= \frac{\pi}{2}$ rad

ANGULAR VELOCITY:

Rate of change of angular displacement of body is called its angular velocity.

ہر سیکنڈ میں باڈی کتنا Angle طے کر رہی ہے۔

Average angular velocity is given as

$$\omega = \frac{total\ angular\ displacement}{total\ time}$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Example1: if a body covers three revolutions in 6 seconds then its angular velocity in SI units will be
(a) 2π rad (a) 4π rad (a) 6π rad (a) π rad

Solution: $\omega = \frac{\Delta\theta}{\Delta t}$
$= \frac{3\times 2\pi}{6} = \pi$

- SI unit of angular velocity is $rads^{-1}$ and other units are $degs^{-1}$ and $revs^{-1}$ etc.
- Angular velocity is a vector quantity and its direction is along axis of rotation determined by right hand rule.
- Angular displacement and angular velocity are always parallel.

Relation with time period

$$\omega = \frac{2\pi\ rad}{T}$$

(اگر کسی Rotating باڈی کا ٹائم پیریڈ معلوم ہو تو اس relation سے ω معلوم کریں)

Angular velocity of second hand of clock	$\omega = \frac{2\pi\ rad}{1\ min} = \frac{2\pi\ rad}{60\ sec}$
Angular velocity of minute hand of clock	$\omega = \frac{2\pi\ rad}{60\ min} = \frac{2\pi\ rad}{3600\ sec}$
Angular velocity of hour hand of clock	$\omega = \frac{2\pi\ rad}{12\times 60\ min} = \frac{2\pi\ rad}{12\times 3600\ sec}$
Angular velocity of earth around its own axis	$\omega = \frac{2\pi\ rad}{1\ day} = \frac{2\pi\ rad}{24\ h}$
Angular velocity of earth around the sun	$\omega = \frac{2\pi\ rad}{1\ year}$

Uniform Angular Velocity:	Instantaneous Angular velocity:
If body covers equal angular displacement in equal intervals of time then body is rotating with uniform angular velocity.	*Angular velocity of a body at any particular instant of time is called instantaneous angular velocity.* $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$

ANGULAR ACCELERATION

Rate of change of angular velocity of a body is called angular acceleration.

Angular acceleration کا مطلب ہے کہ ہر سیکنڈ میں Angular velocity کتنی تبدیل ہو رہی ہے۔

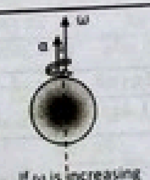
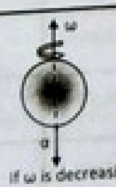
$$\alpha_{av} = \frac{Total\ change\ in\ angular\ velocity}{Total\ time}$$

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t}$$

- Angular acceleration is a vector quantity and its direction is always along the direction of direction of torque.
- SI unit of angular acceleration is $rad s^{-2}$ and other units are $deg s^{-2}$ and $rev s^{-2}$ etc.

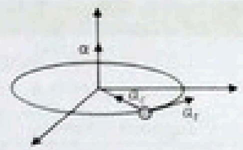
> Linear acceleration is caused by force, similarly angular acceleration is caused by torque
>
> $\tau = I\alpha$

Uniform Angular Acceleration:	Instantaneous Angular Acceleration:
If angular velocity of a body changes equally in equal intervals of time then body is moving with uniform angular acceleration.	Angular acceleration of a body at any particular instant of time is called instantaneous angular acceleration. $\alpha_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$

If angular velocity is increasing then angular acceleration is positive and parallel to angular velocity.	If angular velocity is decreasing then acceleration is negative and anti-parallel to angular velocity.	If angular velocity is constant then angular acceleration is zero and net torque acting on the body is also zero.
 If ω is increasing	 If ω is decreasing	 If ω is constant

Note: A body moving in a circular path may have:

i. **Tangential acceleration** (due to changing speed of the body).
ii. **Angular acceleration** (due to changing angular velocity of the body).
iii. **Centripetal acceleration** (due to changing direction of linear velocity of the body).
iv. a_t, a_c and α are always mutually perpendicular.



If body is moving in circular path with uniform speed or uniform angular velocity then body has only centripetal acceleration due to changing direction of velocity and $\alpha = 0$ and $a_t = 0$

RELATION BETWEEN LINEAR AND ANGULAR VARIABLES

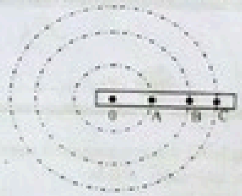
i. $S = r\theta$
ii. $v_t = r\omega$ or $\vec{v}_t = \vec{r} \times \vec{\omega}$ ($\vec{v}_t, \vec{r}$ and $\vec{\omega}$ are always perpendicular to eachother)
iii. $a_t = r\alpha$ or $\vec{a}_t = \vec{r} \times \vec{\alpha}$ ($\vec{a}_t, \vec{r}$ and $\vec{\alpha}$ are always perpendicular to eachother)

Note: For a rotating rigid body. all particles of rigid body will have same angular displacement θ. angular velocity ω and angular acceleration α but values of S, v and a_t may be different depending upon the distance r.

$\theta_A = \theta_B = \theta_C$
$\omega_A = \omega_B = \omega_C$
$\alpha_A = \alpha_B = \alpha_C$

But

$S_C > S_B > S_A$
$v_C > v_B > v_A$
$a_C > a_B > a_A$



Equations of motion for angular motion:

I. Equation:

$$\omega_f = \omega_i + \alpha t$$

without θ کے معلوم کرنا ہو تو 1st equation use کریں

II. Equation:

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

without ω_f کے معلوم کرنا ہو تو 2nd equation use کریں

III. Equation:

$$2\alpha\theta = \omega_f^2 + \omega_i^2$$

without time کے معلوم کرنا ہو تو 3rd equation use کریں

Limitations:

These equations are applicable only if

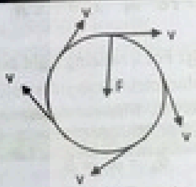
i. Angular acceleration α is uniform.
ii. Axis of rotation does not change.

To determine a relation for angular motion from linear motion replace						
S by θ	v by ω	a by α	m by I	F by τ	p by L	K.E by $K.E_{rot}$

Relation	Linear Motion	Angular Motion
1st Equation of motion	$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
2nd Equation of motion	$S = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$
3rd Equation of motion	$2aS = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 + \omega_i^2$
Newton's 2nd law	$F = ma$ or $F = \frac{\Delta P}{\Delta t}$	$\tau = I\alpha$ or $\tau = \frac{\Delta L}{\Delta t}$
Momentum	$p = mv$	$L = I\omega$
Work done	$W = Fd$	$W = \tau\theta$
Kinetic energy	$K.E = \frac{1}{2}mv^2$	$K.E_{rot} = \frac{1}{2}I\omega^2$

CENTRIPETAL FORCE

If a body is moving in a circular path then direction of its velocity is continuously changing with time. Hence there must be a force perpendicular to velocity that will change the direction of velocity.



یاد رکھیں

باڈی کی direction کو تبدیل کرنے کے لیے force کا ہونا ضروری ہے۔

Example: If force acting on a moving body is zero then its Path or trajectory will be
(a) straight line (b) circular
(c) elliptical (d) parabolic

Solution:
As force is zero so it will not change its direction and move in a Straight line

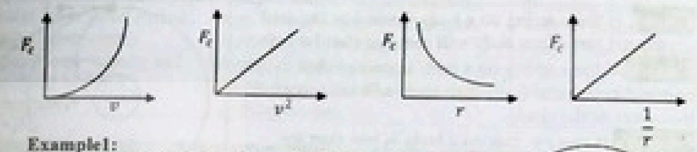
> **"Force needed to bend the normally straight path of a body into circular path is called centripetal force.**
> $$F_c = \frac{mv^2}{r}$$

- Centripetal force is required force provided some agent to bend the body in circular path
- Direction of centripetal force is always directed towards the center but its direction is continuously changing with time .
- Centripetal force is always perpendicular to velocity.
- Centripetal force always changes the direction of velocity it cannot change the speed of the body .

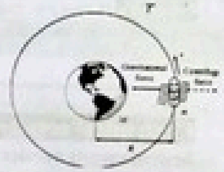
- **Centripetal force depends upon**

• Mass of the body	$F_c \propto m$	For greater mass , greater amount of force is required to bend the body in a circular path
• Speed of the body	$F_c \propto v^2$	With greater speed , greater amount of force is required to bend the body in a circular path
• Radius of circular path	$F_c \propto \frac{1}{r}$	greater amount of force is required to bend the body in a circular path of shorter radius

Various types of graph for centripetal force :

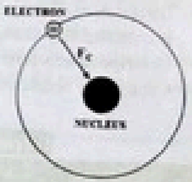


Example1:
Satellites revolving around the earth. Force of gravity provide the required centripetal force.



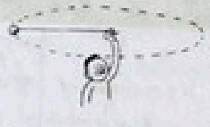
$$F_c = F_g$$
$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$
$$v = \sqrt{\frac{GM}{r}}$$

Example2:
Electrons revolving around the nucleus. Electric force provides the required centripetal force.



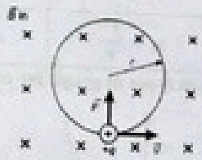
$$F_c = F_e$$
$$\frac{mv^2}{r} = \frac{Kq_1q_2}{r^2}$$

Example3:
A stone tied to a string moving in circular path. Tension in string provide required centripetal force.



$$Fc = T$$

Example4:
A charge moving in circular path in a magnetic field. Magnetic field force provides the required centripetal force.

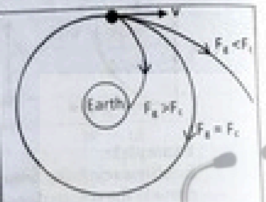


$$F_m = F_c$$
$$qvB = \frac{mv^2}{r}$$
$$v = \frac{qBr}{m}$$

Example5:
A car moving in circular road. Force of friction provides the required centripetal force.

Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide required centripetal force

Case i. If force acting on a body is equal to required centripetal force then body will move in circular path.
Case ii. If force acting on a body is greater than required centripetal force then body will fall towards the center of the circle.
Case iii. If force acting on a body is less than the required centripetal force then body will move out of circular.



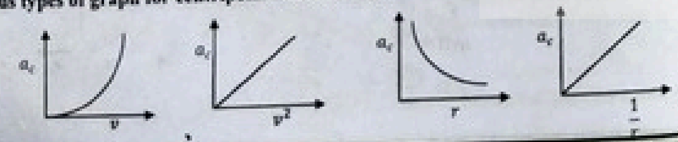
Centripetal acceleration:
Instantaneous acceleration of the body moving in a circular path with uniform speed is always directed towards the center of the circle. It is known as centripetal acceleration.

- Direction of centripetal acceleration is always directed towards the center but its direction is continuously changing with time.
- Centripetal acceleration is always perpendicular to velocity.
- Centripetal acceleration is due to changing the direction of velocity.

Important Expressions for Centripetal Force

In terms of speed	In terms if angular speed	In terms of time period	In terms of momentum	In terms of K.E
$F_c = \frac{mv^2}{r}$ If $v = constant$ $F_c \alpha \frac{1}{r}$	$F_c = mr\omega^2$ If $\omega = constant$ $F_c \alpha r$	$F_c = \frac{4\pi^2 mr}{T^2}$	$F_c = \frac{P^2}{mr}$	$F_c = \frac{2K.E}{r}$
$a_c = \frac{v^2}{r}$	$a_c = r\omega^2$	$a_c = \frac{4\pi^2 r}{T^2}$	$a_c = \frac{P^2}{m^2 r}$	$a_c = \frac{2K.E}{mr}$

Various types of graph for centripetal Acceleration :



Constant quantities	Quantities which are zero	Quantities which are changing direction
Under the action of only centripetal force following quantities remains constant Speed, kinetic energy, **angular speed ,time period , angular momentum, magnitude of velocity and** magnitude of linear momentum	**Under the action of only centripetal force following quantities remains zero.** Work done , change in kinetic energy , tangential acceleration, angular acceleration , tangential force, torque produced by centripetal force, change in angular velocity and change in angular momentum.	**Under the action of only centripetal force magnitude of following remain constant but their direction changes** velocity acceleration, momentum and force.

ORBITAL VELOCITY

Minimum velocity required to put a satellite in a circular orbit is called orbital velocity.

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$

Note: $v \propto \frac{1}{\sqrt{r}}$ and independent of mass of satellite.

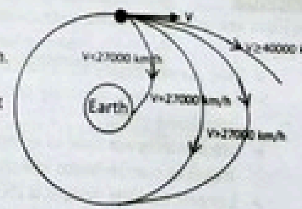
- If satellite is revolving around the earth near its surface then $r = R$

$$v = \frac{GM}{R} = \sqrt{gR} = 7.9\ km/s$$

and $T = 5060\ sec \approx 84\ min$

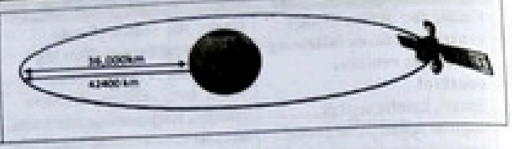
Minimum height of satellite revolving around the earth is 400 km and 24 such satellites form GPS system.

- If velocity of satellite is less than **critical velocity** ($v < 27000 km/h$) it will fall towards earth.
- **If velocity is equal to critical velocity** ($v = 27000 km/h$) then it will move in circular path.
- **If velocity is greater than critical velocity** but less than escape velocity it will move in an elliptical path.
- **If velocity is equal or greater than escape velocity** it will escape from earth's gravity



GEOSTATIONARY SATELLITE

"Satellite whose orbital motion is synchronized with the rotation of earth is known as geostationary satellite. And its orbit is known as geostationary orbit".



Orbital speed	Radius of orbit	Height
$v = \sqrt{\frac{GM}{r}}$	$r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$	$h = r - R$
$v = 3.1\ km/s$	$v = 4.23 \times 10^4\ km$	$h \approx 36000\ km$

Time period	Angular velocity	Angular acceleration
T	$\omega = \frac{2\pi}{T}$	$\alpha = \frac{\Delta\omega}{\Delta t}$
$T = 1day$ $= 24h$ $= 86400\ sec$	$\omega = \frac{1rev}{day}$ $= \frac{2\pi\ rad}{day}$	$Zero$

Applications:

Such satellites are useful for worldwide communication, weather observation, navigation and military uses.

- One geostationary satellite can cover 120° longitude of earth.
- Minimum three correctly positioned satellites are required for complete coverage of populated earth.
- Microwaves are used to communicate with geostationary satellites because they travel in narrow beam and pass easily through atmosphere.
- Largest satellite system is INTELSAT (International Telecommunication Satellite Organization) managed by 126 countries.
- INTELSAT IV has capacity of 30,000 two way telephone calls plus 3 T.V channels and its operates at 4,6,11 and 14 GHz frequencies.

UNIT 04 〉〉 OSCILLATIONS & WAVES

OSCILLATIONS

Periodic Motion:

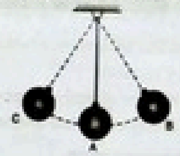
Motion which repeats itself after regular intervals of time is called periodic motion.

Vibratory Motion:

To and fro motion of a body about its mean position is called vibratory motion.

Examples:

- Motion of simple pendulum.
- Motion of mass spring system.
- Motion of tuning fork.
- Motion of atoms in solids.



Amplitude:

Maximum displacement covered by body from its mean position is called amplitude.

Vibration:

One complete round trip of a body in motion is called one vibration.

Time period:

Time taken by body to complete one vibration is called time period .

Frequency:

Number of vibrations executed by a body in one second is called frequency.

$f = \frac{1}{T}$

Product of time period and frequency is always equal to one

Angular Frequency:

Number of revolutions executed by a body in one second is called angular frequency.

$\omega = \frac{2\pi}{T}$ $\omega = 2\pi f$

Example: Angular frequency of second pendulum is

(a) $2\pi\ rev\ s^{-1}$
(b) $2\pi\ Hz$
(c) $\pi\ Hz$
(d) $2\pi\ rad\ s^{-1}$

Solution: As time period of simple pendulum is 2sec

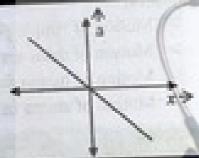
$$\omega = \frac{2\pi}{2} = \pi rad\ s^{-1}$$

No. of vibrations	Distance	Displacement	Time taken	Move from
$\frac{1}{4}$ vib	x_o	x_o	$\frac{T}{4}$	Mean to extreme
$\frac{1}{2}$ vib	$2x_o$	Zero	$\frac{T}{2}$	Mean to extreme
$\frac{3}{4}$ vib	$3x_o$	x_o	$\frac{3T}{4}$	Mean to left extreme
1vib	$4x_o$	Zero	T	Mean to mean

SIMPLE HARMONIC MOTION

Simple harmonic motion has following characteristics:

i. Always vibratory motion.
ii. Acceleration is always directly proportional to displacement.
iii. Acceleration is always directed towards mean position.



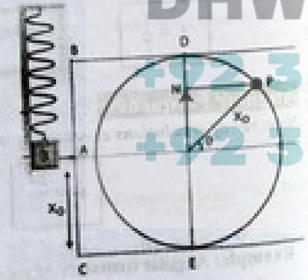
$$a \propto -x$$

Note
The graph between acceleration and displacement is a straight line in II and IV quadrant.

Examples:
Mass spring system executes SHM.
Simple pendulum is executing SHM for small amplitude.

SHM & UNIFORM CIRCULAR MOTION

- Mass spring system executes SHM with amplitude X_o and time period T.
- If point P is moving in circular path with uniform speed or uniform angular velocity then its periodic but not SHM.
- Projection N of the point P is oscillating on vertical axis and is executing SHM.



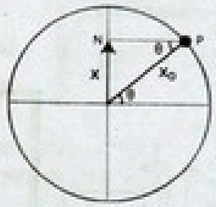
Example : Projection of a point on any diameter executes simple harmonic motion if the point is moving in a circular path with
(a) uniform speed ✓ (a) variable speed (a)uniform acceleration (d) none of these

Instantaneous displacement:

From the figure:

$$sin\theta = \frac{x}{x_o}$$

$$x = x_o\ sin\theta$$



Types of Questions:

1. **Find displacement when θ is given OR Find $\theta(phase)$ when displacement is given.**

Example1:
What is displacement of a body executing SHM when its phase is $\frac{\pi}{6}$ rad.
(a) $\frac{x_o}{2}$ (b) $\frac{x_o}{\sqrt{2}}$ (a) $\frac{\sqrt{3}x_o}{2}$ (d) x_o

Solution:
$x = x_o\ sin\theta$
$x = x_o sin\frac{\pi}{6} = x_o\ sin30^o$
$= \frac{x_o}{2}$

Example2:
If a body is executing SHM then find the value of θ for which displacement is 70% of maximum displacement.
(a)30° (b) 45° (c) 60° (d) 70°

Solution:
$x = x_o\ sin\theta x = 0.7x_o = \frac{x_o}{\sqrt{2}}$
$\Rightarrow Sin\ \theta = \frac{1}{\sqrt{2}}$
$\Rightarrow \theta = 45^o = \frac{\pi}{4} rad$

2. **Find displacement when time is given OR Find the time when displacement is given.**

Example:
What is displacement of body at instant $t = \frac{T}{8}$ where T is time period of the a body executing SHM.
(a)30° (b) 45° (c) 60° (d) 70°

Solution:
$t = \frac{T}{8} \Rightarrow \theta = \frac{\pi}{4} = 45^o$
$x = x_o\ sin\theta$
$\Rightarrow x = \frac{x_o}{\sqrt{2}}$

Alternate Solution
$x = x_o\ sin\theta = x_o\ sin\omega t = x_o\ sin\frac{2\pi}{T} \times \left(\frac{T}{8}\right)$
$= x_o\ sin\frac{\pi}{4} = x_o\ sin45^o$
$= \frac{x_o}{\sqrt{2}}$

SHORT CUT
Time سے θ معلوم کرنے کے لیے T کو
2π کے برابر رکھ کر half کر دیں

Example:
The time taken by body executing SHM from its mean position to half of its extreme position.
(a) $\frac{T}{4}$ (b) $\frac{T}{6}$ (c) $\frac{T}{8}$ (d) $\frac{T}{12}$

Solution:
$$x = x_o \sin\theta$$
$$x = \frac{x_o}{2} \Rightarrow \theta = 30^\circ = \frac{\pi}{6}$$
$$\Rightarrow t = \frac{T}{12}$$

Alternate Solution:
$$x = \frac{x_o}{2} \Rightarrow x_o \sin\theta = \frac{x_o}{2} \Rightarrow \theta = \frac{\pi}{6}$$
$$\Rightarrow \omega t = \frac{\pi}{6} \Rightarrow \frac{2\pi}{T} t = \frac{\pi}{6} \Rightarrow \quad \boxed{t = \frac{T}{12}}$$

SHORT CUT
θ سے Time کی ویلیو معلوم کرنے کیلئے π کی جگہ T لگا دیں
پھر ویلیو کو double کر دیں

Example:
The time taken by body executing SHM from its extreme position to a point midway between mean and extreme position.
(a) $\frac{T}{4}$ (b) $\frac{T}{6}$ (c) $\frac{T}{8}$ (d) $\frac{T}{12}$

Solution:
$$x = x_o \cos\theta$$
$$x = \frac{x_o}{2} \Rightarrow \theta = 60^\circ = \frac{\pi}{3}$$
$$\Rightarrow t = \frac{T}{6}$$

Alternate Solution:
$$x = \frac{x_o}{2} \Rightarrow x_o \sin\theta = \frac{x_o}{2} \Rightarrow \theta = \frac{\pi}{6}$$
$$\Rightarrow \omega t = \frac{\pi}{6} \Rightarrow \frac{2\pi}{T} t = \frac{\pi}{6} \Rightarrow \quad \boxed{t = \frac{T}{12}}$$

When motion starts from extreme position then use the relation
$$x = x_o \cos\theta$$

3. Comparison type questions.

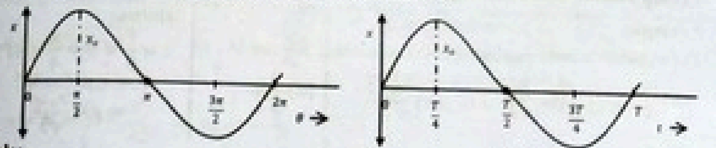
$$x = x_o \sin(\omega t + \phi)$$
(x_o: Amplitude; ω: Angular frequency; ϕ: Initial phase)

Example: Displacement for a body executing SHM is given as $x = 10\sin(4t)$. Find amplitude, maximum velocity, maximum acceleration, angular frequency, time period, frequency and initial phase.

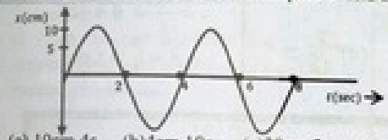
Solution:
Compare the equation $x = 10\sin(4t)$ with standard equation $x = x_o \sin(\omega t + \phi)$

Amplitude $= x_o = 10$
Maximum velocity $= x_o\omega = (10)(4) = 40$
Time period $= T = \frac{2\pi}{\omega} = \frac{\pi}{2}$
Max. acceleration $= x_o\omega^2 = (10)(4)^2 = 160$
Angular frequency $= \omega = 4$
Initial phase $= 0$
Frequency $= f = \frac{1}{T} = \frac{2}{\pi}$

Graph: The graph between x and θ, or x and time is sinusoidal

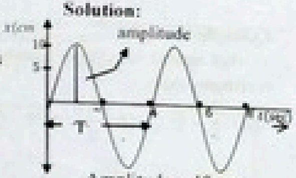


Example:
Displacement time graph of a body executing SHM is shown in the figure below. The value of amplitude and time period is



(a) 10cm,4s (b)4cm,10s (c)20cm,2s (d)20cm,4s

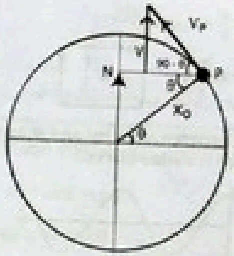
Solution:



Amplitude = 10cm
T = 4s

Instantaneous Velocity:

Velocity of point P is always directed along tangent to the circle and ventricle component of its velocity is the velocity of projection which is executing SHM.
From the figure:



$$\sin(90 - \theta) = \frac{v}{v_p}$$
$$\cos\theta = \frac{v}{v_p}$$
$$v = v_p$$
$$\boxed{v = x_o\omega \cos\theta}$$

Types of Questions:

1. **Find velocity when θ is given OR find θ when velocity is given.**

Example:
What is velocity of the body executing SHM when its phase angle is 30°.
(a) $\frac{v_o}{2}$ (b) $\frac{v_o}{\sqrt{2}}$ (c) $\frac{v_o\sqrt{3}}{2}$ (d) $\frac{v_o}{\sqrt{3}}$

Solution:
$$v = x_o\omega \cos\theta$$
$$v = x_o\omega \cos 30^\circ$$
$$= \frac{x_o\omega\sqrt{3}}{2} = \frac{v_o\sqrt{3}}{2}$$

Example:
The value of θ for which velocity of a body executing SHM is $\frac{v_o}{2}$
(a) 30° (b) 45° (c) 60° (d) 90°

Solution:
$$v = x_o\omega \cos\theta$$
$$v = \frac{V_o}{2} \Rightarrow \theta = 60^\circ = \frac{\pi}{3}$$

2. Find velocity when time is given OR find time when velocity is given.

Example:
The velocity of body executing SHM at instant $t = \frac{T}{8}$ sec

(a) $\frac{v_o}{2}$ (b) $\frac{v_o}{\sqrt{2}}$ (c) $\frac{v_o\sqrt{3}}{2}$ (d) $\frac{v_o}{\sqrt{3}}$

Solution:
$t = \frac{T}{8} \Rightarrow \theta = \frac{\pi}{4} = 45°$
$v = x_o\omega\, Cos45°$
$\Rightarrow v = \frac{x_o\omega}{\sqrt{2}}$

Example:
At what instant velocity of a body executing SHM is half of its maximum value.

(a) $\frac{T}{4}$ (b) $\frac{T}{6}$ (c) $\frac{T}{8}$ (d) $\frac{T}{12}$

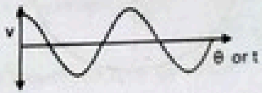
Solution:
$v = x_o\omega\, Cos\theta$
$v = \frac{v_o}{2} \Rightarrow \theta = 60°$
$\Rightarrow \theta = \frac{\pi}{3} \Rightarrow t = \frac{T}{6}$

Alternate Solution:
$v = \frac{v_o}{2} \Rightarrow v_o cos\theta = \frac{v_o}{2} \Rightarrow cos\theta = \frac{1}{2}$
$\Rightarrow \theta = \frac{\pi}{3} \Rightarrow \omega t = \frac{\pi}{3}$
$\Rightarrow \frac{2\pi}{T}t = \frac{\pi}{3} \Rightarrow$ $t = \frac{T}{6}$

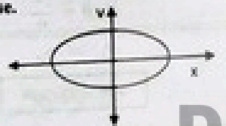
SHORT CUT

θ سے Time کی ویلیو معلوم کرنے کیلئے π کی جگہ T اور نیچے والی ویلیو کو double کر دیں۔
Time سے θ کی ویلیو معلوم کرنے کیلئے T کی جگہ π اور نیچے والی ویلیو کو half کر دیں۔

Graph between velocity and θ or velocity and time is cosine curve.



Graph between velocity and displacement is an ellipse.



3. Comparison type questions.

$v = v_o cos(\omega t + \phi)$
v_o → Max velocity; ω → Angular frequency; ϕ → Initial phase

Example: velocity a body executing SHM is given as $v = 10\cos 2t$. Find amplitude, maximum velocity, maximum acceleration, angular frequency, time period, frequency and initial phase.
Solution: compare the equation $v = 10\cos 2t$ with the standard equation $v = v_o cos(\omega t + \phi)$

i. $v_o = 10$	ii. $\omega = 2$
iii. $x_o = \frac{v_o}{\omega} = \frac{10}{2} = 5$	iv. $a_o = x_o\omega^2 = 5(2)^2 = 20$
v. $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$	vi. $T = \frac{1}{f} = \frac{1}{\pi}$

4. Velocity In terms of displacement:

$v = \omega\sqrt{x_o^2 - x^2} = \omega\sqrt{x_o^2 - \frac{x^2x_o^2}{x_o^2}}$

$v = x_o\omega\sqrt{1 - \frac{x^2}{x_o^2}} = v_o\sqrt{1 - \frac{x^2}{x_o^2}}$

- At mean position $x = 0$
  $\Rightarrow v = x_o\omega$ (maximum)
- At mean position $x = x_o$
  $\Rightarrow v = 0$ (minimum)

➤ Speed of projection increases when it is moving towards center of the circle and its speed decreases when it is moving away from the center of circle

Example:
If a body is executing SHM then at what displacement velocity is half of its maximum velocity.

(a) $\frac{v_o}{2}$ (b) $\frac{v_o}{\sqrt{2}}$ (c) $\frac{v_o\sqrt{3}}{2}$ (d) $\frac{v_o}{\sqrt{3}}$

Solution:
$v = \omega\sqrt{x_o^2 - x^2} \Rightarrow \frac{x_o\omega}{2} = \omega\sqrt{x_o^2 - x^2}$
$\Rightarrow \frac{x_o}{2} = \sqrt{x_o^2 - x^2} \Rightarrow \frac{x_o^2}{4} = (x_o^2 - x^2)$
$\Rightarrow x^2 = x_o^2 - \frac{x_o^2}{4} \Rightarrow x^2 = \frac{3x_o^2}{4}$
$x = \frac{\sqrt{3}x_o}{2}$

Alternate short cut solution:

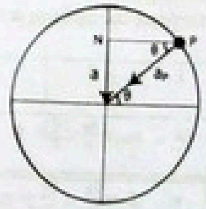
Velocity سے displacement کو direct معلوم کرنے کے لئے صرف درج ذیل تبدیلیاں کریں۔

As
$v = \frac{v_o}{2}$
$\Rightarrow x = \frac{x_o\sqrt{3}}{2}$

➤ Replace v_o by x_o and vice versa.
➤ Replace $\frac{1}{2}$ by $\frac{\sqrt{3}}{2}$ and vice versa.
➤ Replace 0 by 1 and vice versa.
➤ Replace $\frac{1}{\sqrt{2}}$ by $\frac{1}{\sqrt{2}}$

Instantaneous Acceleration:

➤ Since point P is moving in circular path with uniform speed hence it only posses centripetal acceleration directed towards the center of the circle.
➤ Its vertical component is acceleration of projection N executing SHM. From the figure:



$sin\theta = \frac{a}{a_p}$ or $a = a_p\, sin\theta$

$a = x_o\omega^2 sin\theta$ $\vec{a} = -\omega^2\vec{x}$ $a = x_o\omega^2 sin\omega t$

1. **Find acceleration when θ is given OR find θ when acceleration is given.**

Example:
Acceleration of a body executing SHM at angle 30° is
(a) $\frac{\sqrt{3}a_o}{2}$ (b) $\frac{a_o}{\sqrt{2}}$ (c) $\frac{a_o}{2}$ (d) a_o

Solution:
$a = x_o\omega^2 \sin 30°$
$a = \frac{x_o\omega^2}{2} = \frac{a_o}{2}$

Example:
For which of θ acceleration of a body executing SHM is $\frac{x_o\omega^2}{\sqrt{2}}$
(a) 30° (b) 45° (c) 60° (d) 90°

Solution:
$a = x_o\omega^2 \sin\theta$
$a = \frac{x_o\omega^2}{\sqrt{2}} \Rightarrow \theta = 45°$

2. **Find time when acceleration is given and vice versa.**

Example:
Acceleration of a body executing SHM at instant $t = \frac{T}{6}$ is
(a) $\frac{\sqrt{3}a_o}{2}$ (b) $\frac{a_o}{\sqrt{2}}$ (c) $\frac{a_o}{2}$ (d) a_o

Solution:
$t = \frac{T}{6} \Rightarrow \theta = \frac{\pi}{3} = 60°$
$a = x_o\omega^2 \sin\theta$
$\Rightarrow a = \frac{x_o\omega^2\sqrt{3}}{2} = \frac{a_o\sqrt{3}}{2}$

Example:
At what instant the acceleration of the body is $\frac{x_o\omega^2}{2}$ or $\frac{a_o}{2}$
(a) $\frac{T}{4}$ (b) $\frac{T}{6}$ (c) $\frac{T}{8}$ (d) $\frac{T}{12}$

Solution: $a = x_o\omega^2 \sin\theta$
$a = \frac{a_o}{2} \Rightarrow \theta = 30°$
$= \frac{\pi}{6} \Rightarrow t = \frac{T}{12}$

3. **Comparison type questions:**

$a = a_o \sin(\omega t + \phi)$
- a_o: Max. acceleration
- ω: Angular frequency
- ϕ: Initial phase

Example: acceleration of a body executing SHM is given as $a = 8\sin 2t$. Find amplitude, maximum velocity, maximum acceleration, angular frequency, time period, frequency and initial phase.

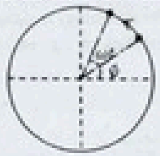
Solution :
Compare the equation $a = 8\sin 2t$ with the standard equation $a = a_o\sin(\omega t + \phi)$

i. $a_o = 8$	ii. $\omega = 2$
iii. $v_o = \frac{a_o}{\omega} = \frac{8}{2} = 4$	iv. $x_o = \frac{a_o}{\omega^2} = \frac{8}{4} = 2$
v. $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$	vi. $T = \frac{1}{f} = \frac{1}{\pi}$

PHASE

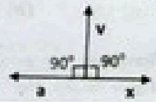
Angle of θ which specifies the displacement as well as direction of motion of a body executing SHM is called phase.

- Initial phase at $t = 0 = \emptyset$
- phase during the time $t = \omega t$
- **Total phase = $\omega t + \emptyset$**



Motion starts t = 0 from	Initial phase	Displacement	Velocity	Acceleration
....	$\emptyset$	$x = x_o\sin(\omega t + \emptyset)$	$v = x_o\omega\cos(\omega t + \emptyset)$	$a = -x_o\omega^2\sin(\omega t + \emptyset)$
Mean position	$\emptyset = 0$	$x = x_o\sin\omega t$	$v = x_o\omega\cos\omega t$	$a = -x_o\omega^2\sin\omega t$
Extreme position	$\emptyset = 90°$	$x = x_o\cos\omega t$	$x = -x_o\sin\omega t$	$a = -x_o\omega^2\cos\omega t$

Note:
- Phase difference between displacement and velocity is 90°.
- Phase difference between velocity and acceleration is 90°.
- Phase difference between displacement and acceleration is 180°.



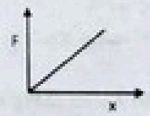
MASS SPRING SYSTEM

- Mass spring system executes simple harmonic motion.
- Restoring force brings the body **back towards mean position.**
- Body does not come to rest at mean position due to inertia.

Hooke's Law: ***Applied force is directly proportional to extension produced in spring.***

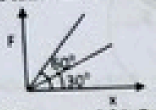
$F \propto x$ OR $F = kx$

- The graph between force and extension is a straight line.
- **Slope of the graph represents the spring constant.**
- **Area under force-extension graph represents the work done or P.E.**



1. [illegible] slope [illegible] spring constant [illegible]
2. [illegible] Area [illegible] P.E [illegible] work done [illegible]

Example:
Force-extension graph for two different springs is shown in the figure below. Find the ratio between their spring constants.



(a) $1:\sqrt{3}$ (b) $\sqrt{3}:1$ (c) $1:3$ (d) $3:1$

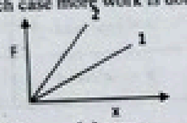
Solution:

$K = slope = Tan\theta$

$\frac{K_1}{K_2} = \frac{Tan30^o}{Tan60^o}$

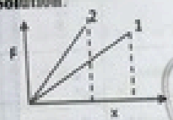
$= \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}$

Example:
Force-extension graph for two different springs is shown in the figure below. In which case more work is done.



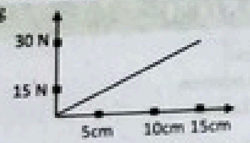
(a) 1st (b) 2nd (c) same (d) zero in both

Solution:



$(W_1 > W_2)$

Example: Force-extension graph for a spring is shown in the figure below. The value of spring constant and P.E stored in the spring



Solution:

$K = \frac{F}{x} = \frac{30N}{15cm}$

$= \frac{30N}{15\times10^{-2}m} = 200\ Nm^{-1}$

$W = Area$

$= \frac{1}{2}(15\times10^{-2})(30) = 2.25J$

Spring Constant or Force constant :

Ratio of applied force to extension produced in spring is known as spring constant.

- $K = \frac{F_{ext}}{x}$
- SI unit = $Nm^{-1} = kgms^{-2}$
- Spring constant and surface tension have same units.
- Spring constant depends upon temperature and nature of the material.
- Spring constant is independent of applied force and extension produced in spring.

Example: A mass of 5kg suspended with a vertical spring produces 2cm extension in the spring . The spring constant of the spring will be

(a) 500 Nm^{-1} (b) 5000 Nm^{-1} (c) 2500 Nm^{-1} (d) 250 Nm^{-1}

Solution:

$k = \frac{F}{x}$

$= \frac{mg}{x} = \frac{5\times10}{2\times10^{-2}} = 2500\ Nm^{-1}$

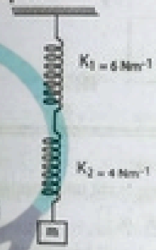
Series Combination of Springs

If springs are connected end to end this combination is known as series combination

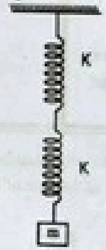
Equivalent Spring Constant

1. $K_{eq} = \frac{K}{n}$ — 1. (اگر "n" different value والے سپرنگ سیریز میں لگے ہوں)
2. $K_{eq} = \frac{K_1K_2}{K_1+K_2} = \frac{Product}{sum}$ — 2. (اگر دو different value والے سپرنگ سیریز میں لگے ہوں)
3. $\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \cdots$ — 3. (اگر different value والے سپرنگ سیریز میں لگے ہوں)

Example:1



Example:2



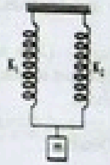
$K_{eq} = \frac{K}{n} = \frac{K}{2}$

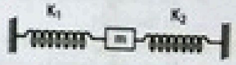
NOTE:
Equivalent spring constant series is less than minimum spring constant.
And to decrease the spring constant springs are connected in series

Parallel Combination of Springs

If springs are connected side by side then this combination is known as parallel combination

[illegible] spring [illegible] end [illegible] end [illegible]



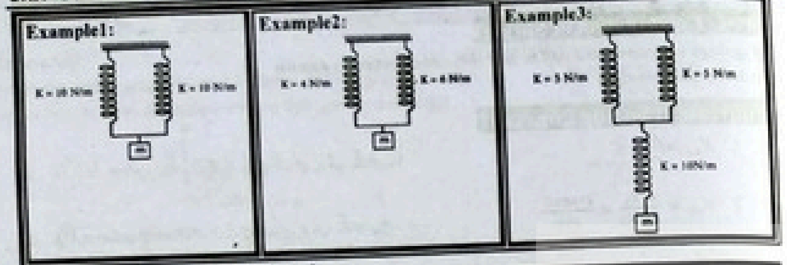


Equivalent Spring Constant

If springs are connected side by side then equivalent spring constant is given as :

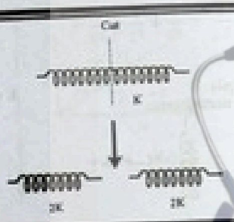
1. $K_{eq} = nK$ (If "n" no. of springs having same value are connected in parallel)
2. $K_{eq} = K_1 + K_2 + \cdots$ (If springs having different values are connected in parallel)
3. $K_{eq} = K_{max}$
4. To increase spring constant springs are connected in parallel.

Example1: **Example2:** **Example3:**



Note:
If a spring of spring constant K is cut into '*n*' equal parts then spring constant of each part will be 'nK'

Example:
If spring of spring constant K is cut into two equal parts then spring constant of each part will be 2K.



Examples: If spring is cut into two parts having length in the ratio 1:2 then the ratio between spring constants of the part will be.

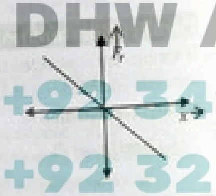
a) 1:2 ✓b) 2:1 c) 1:4 d) 4:1

Solution:
If spring is cut into unequal parts then $\left(K \propto \frac{1}{l}\right)$
ratio in k will be opposite to *l*

Restoring Force:

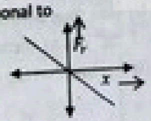
Force which brings the body back towards its mean position.

- Restoring force is the force which produces acceleration in the body.
- $F_r = -F_{ext} = -Kx = -m\omega^2 x \quad (K = m\omega^2)$
- Restoring force is always directed towards mean position and is opposite to displacement.
- Graph between restoring force and displacement is a straight line in 2nd and 4th Quadrant.



Example: The graph between restoring force and displacement is shown in the figure . Slope of the graph is directly proportional to

(a) ω (b) ω^2
(b) $1/\omega$ (d) $1/\omega^2$



Solution:
As $F_r = -m\omega^2 x$

$$slope = \frac{F_r}{x} = -m\omega^2$$
$$slope \propto \omega^2$$

Acceleration:

As
$$F_r = -Kx$$
$$ma = -Kx$$
$$a = -\frac{K}{m}x$$
$$max.\,acceleration = -\frac{K}{m}x_o$$
$$\Rightarrow a \propto -x$$

Hence mass spring system is executing simple harmonic motion.

Angular Frequency:

As $a = -\frac{K}{m}x$

$$-\omega^2 x = -\frac{K}{m}x \ (\because a = -\omega^2 x)$$
$$\omega = \sqrt{\frac{K}{m}}$$

Note:
Angular frequency only depends upon spring constant and mass suspended and independent of amplitude and gravity.

Time Period:

As $T = \frac{2\pi}{\omega}$

or
$$T = 2\pi\sqrt{\frac{m}{K}}$$
$$T \propto \sqrt{m} \quad and \quad T \propto \frac{1}{\sqrt{K}}$$

Note: Time period only depends upon spring constant and mass suspended and independent of amplitude and gravity.

Frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$
$$f \propto \sqrt{K} \quad and \quad f \propto \frac{1}{\sqrt{m}}$$

Note: Frequency only depends upon spring constant and mass suspended and independent of amplitude and gravity.

If spring is being vertically then we may write $F = mg$

$$\Rightarrow Kx = mg \Rightarrow \frac{m}{K} = \frac{x}{g}$$
$$T = 2\pi\sqrt{\frac{x}{g}} \quad and \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{x}}$$

Since now $x \propto g$ hence time period and frequency are still independent of gravity.

Example: If a 2kg mass suspended with a vertical spring produces 10cm extension in the spring . If it is set into oscillations its time period will be

(a) ω (b) ω^2
(b) $1/\omega$ (d) $1/\omega^2$

Solution: $T = 2\pi\sqrt{\frac{x}{g}}$
$$= 2\pi\sqrt{\frac{10}{10 \times 100}} = \frac{\pi}{5}$$

Displacement:

Instantaneous displacement for mass spring system is given as

$$x = x_o \sin\omega t$$

$$\Rightarrow x = x_o \sin\sqrt{\frac{K}{m}}\,t$$

$$max.\,displacement = x_o$$

Velocity:

$$v = \omega\sqrt{x_o^2 - x^2} = \sqrt{\frac{k}{m}}\sqrt{x_o^2 - x^2}$$

$$= \sqrt{\frac{k}{m}(x_o^2 - x^2)} = \sqrt{\frac{k}{m}}\,x_o\sqrt{1 - \frac{x^2}{x_o^2}}$$

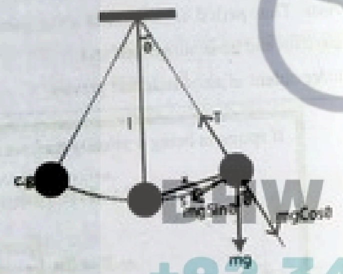
$$v = v_o\sqrt{1 - \frac{x^2}{x_o^2}}$$

$$max.\,velocity \quad v_o = x_o\omega = x_o\sqrt{\frac{k}{m}}$$

SIMPLE PENDULUM

Simple pendulum consists of small heavy mass suspended with a light and intensible string whose other end is fixed with rigid support.

ℓ = length of pendulum
θ = Angular displacement with vertical axis
s = distance from mean position
x = displacement from mean position
mg = weight
$mg\cos\theta$ = component along the string
$mg\sin\theta$ = component perpendicular to the string



Tension in String:

Since no acceleration is produced along the string hence net force along the string is zero.

$$T - mg\cos\theta = 0$$

$$T = mg\cos\theta$$

- At mean position ($\theta = 0$) tension is maximum and equal to weight $T = mg\cos = mg$
- At extreme position ($\theta = maximum$) Tension is maximum.
- If angular displacement θ is very small then $\cos\theta \approx 1$ and $T \approx mg \approx$ constant.

Restoring Force:

The component $mg\sin\theta$ brings the body back towards its mean position

$$F_r = -mg\sin\theta$$

- -ve sign indicates that it is directed towards the mean position.
- **Torque acting on pendulum is $mg\ell\sin\theta$**

If pendulum makes an angle θ with the horizontal instead of vertical then

Tension in string is

$$T = mg\cos\theta$$

Restoring force is

$$F_r = -mg\cos\theta$$

Acceleration

$$ma = -mg\sin\theta$$

$$a = -g\sin\theta$$

- If angle θ is very small
- $\sin\theta \approx \theta \approx \frac{x}{\ell}$ $\quad \left(\because \theta = \frac{x}{\ell}\right)$

$$a = -g\frac{x}{\ell}$$

OR $\quad a \propto -x$

Simple pendulum executes SHM for small angular displacement 'θ'.

Angular Frequency:

$$a = -g\frac{x}{\ell}$$

$$-\omega^2 x = \frac{g}{\ell}x \Rightarrow \omega^2 = \frac{g}{\ell}$$

$$\omega = \sqrt{\frac{g}{\ell}}$$

$$\omega \propto \sqrt{g} \text{ and } \omega \propto \frac{1}{\sqrt{\ell}}$$

Note:

(i) Angular frequency only depends upon length of pendulum and acceleration due to gravity.

(ii) It is independent of amplitude and mass of the bob.

Time Period

As $T = \frac{2\pi}{\omega}$

Or

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$\Rightarrow T \propto \sqrt{\ell}$ and $T \propto \frac{1}{\sqrt{g}}$

Note:

- T only depends upon length and acceleration due to gravity.
- T is independent of amplitude and mass of the body.

Frequency:

As $f = \frac{1}{T}$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

$\Rightarrow \quad T \propto \sqrt{g} \quad and \quad T \propto \frac{1}{\sqrt{\ell}}$

Note:

- Frequency only depends upon length and acceleration due to gravity.
- Frequency is independent of amplitude and mass of the body.

Second Pendulum:
A pendulum whose time period is two seconds is called second pendulum.
$T = 2sec.$ $f = 0.5\ Hz.$ $\ell = 99.3$ cm (when $a = 9.8ms^{-2}$)

Example1: If time period of second pendulum on surface of earth is two seconds. Then time period of second pendulum on surface of moon will be
(a) 2sec (b) $2\sqrt{6}$ sec (c) $2/\sqrt{6}$ sec (d) infinity

Solution: Two second
[illegible] second pendulum [illegible]
[illegible] 2 sec [illegible]

Example2:
If time period of second pendulum on earth is two seconds and it is shifted to moon then its time period will be (value of $g_{moon} = \frac{g_{earth}}{6}$)
(a) 2sec (b) $2\sqrt{6}$ sec (c) $2/\sqrt{6}$ sec (d) infinity

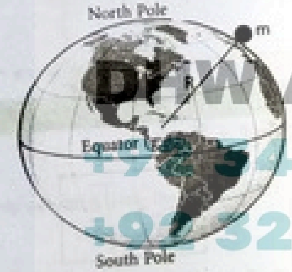
Solution:
$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T \propto \frac{1}{\sqrt{g}}$
As $g' = \frac{g}{6} \Rightarrow T' = \sqrt{6}\,T = 2\sqrt{6}\ s$

[illegible] pendulum [illegible] shift [illegible] pendulum [illegible] change [illegible]

VARIATION IN VALUE OF 'g'

On surface of earth

- Expression for 'g' on the surface of earth is $g = \frac{GM}{R^2}$
- **At poles value of g is maximum** due to shorter distance from center of earth and value of g is independent of rotation of earth.
- **At equator value of g is maximum** due to longer distance from center of earth and due to rotation of earth. (value of g varies inversely with rotation of earth. If angular velocity of earth increases value of g decreases and vice versa).
- By moving from poles to equator value of 'g' decreases.
- By moving from equator to pole value of 'g' increases.



1. Above the surface of earth:

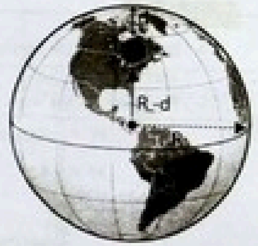
- At height h value of 'g' is given as $g' = \frac{GM}{(R+h)^2}$
- By increasing the height value of 'g' decreases.

$g_{karachi} > g_{murree} > g_{(K-2)}$

Height	$h = \frac{R}{2}$	$h = R$	$h = 2R$	$h = 3R$
g	$g' = \frac{4g}{9}$	$g' = \frac{g}{4}$	$g' = \frac{g}{9}$	$g' = \frac{g}{16}$

2. In depth of earth:

- At depth 'd' value of 'g' is given as
$$g' = \frac{GM(R-d)}{R^3}$$
- By increasing depth value of 'g' decreases.



Example: If $d = \frac{R}{2}$

$$g' = \frac{GM\left(R - \frac{R}{2}\right)}{R^3}$$
$$= \frac{GM\left(\frac{R}{2}\right)}{R^3} = \frac{GM}{2R^2} = \frac{g}{2}$$

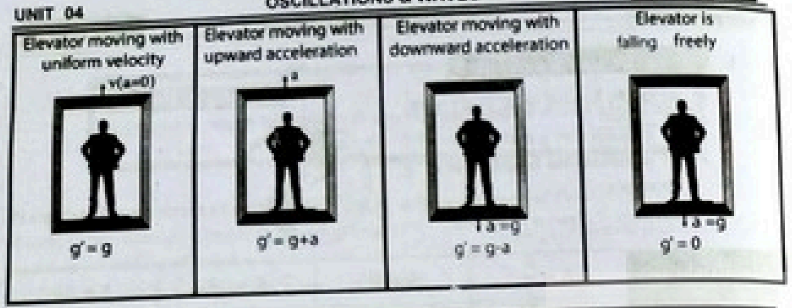
3. Value of 'g' is zero in four cases:

i. At center of the planets.
ii. At infinite distance from the planet.
iii. Inside satellites revolving around the earth.
iv. Inside a freely falling system.

(At these places time period of pendulum is infinity and frequency will be zero)

4. In an accelerating frame of reference

i. If it is moving upward with acceleration 'a' then $g' = g + a$ (g increases).
ii. If it is moving downward with acceleration 'a' then $g' = g - a$ (g decreases).
iii. If it is at rest or moving with uniform velocity ($a = 0$) then $g' = g$.
iv. If it is moving in horizontal direction with acceleration a then $g' = \sqrt{g^2 + a^2}$ (g increases)



ENERGY CONVERSION IN SHM

➢ ***If a body is executing SHM then P.E and K.E interchanges into each other but total amount of energy always remains constant.***

Work done:

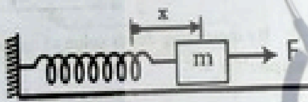
➢ In stretching the spring work is done by variable force.

➢ Expressions for work done.

i. $W = \frac{1}{2}Fx$

ii. $W = \frac{1}{2}kx^2$

iii. $W = \frac{F^2}{2k}$



Questions میں دی گئی information کو دیکھ کر expression کو decide کریں۔

Example1:
If two different springs are subjected to same amount of force and the ratio between their extensions is 1:2 then the ratio between the work done will be.
(a) 1:2 (b) 2:1 (c) 1:4 (d) 4:1

Solution:
$\Rightarrow \quad W \propto x \quad (F = constant)$
So ratio between work is 1:2

Example 2:
To produce 2cm extension in the spring 5J work is done on it. How much work is required to produce 4cm extension?
(a) 5J (b) 10J (c) 20J (d) 40J

Solution:
$W = \frac{1}{2}kx^2 \Rightarrow$
$W \propto x^2 \ (k = constant)$
Since extension is doubled so work becomes 4 times $W = 4(5) = 20J$

Example3:
If 2N force is applied on a spring having spring constant $10Nm^{-1}$ then work done is equal to.
(a) 1:2 (b) 2:1 (c) 1:4 (d) 4:1

Solution:
$W = \frac{F^2}{2k} = \frac{(2)^2}{2 \times 10}$
$= \frac{2}{10} = 0.2J$

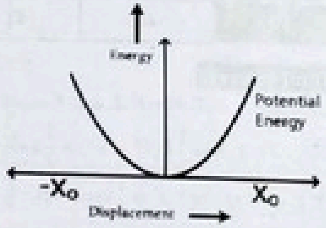
Potential Energy:

➢ Since elastic force is conservative force so work done is equal to gain in P.E.

$P.E = \frac{1}{2}Fx, \quad P.E = \frac{1}{2}kx^2, \quad P.E = \frac{F^2}{2k}$

➢ At mean position $x = 0$
$P.E = 0 \ (minimum)$

➢ At extreme position $x = x_o$
$P.E = \frac{1}{2}kx_o^2 \ (maximum)$



Displacement	$x = 0$	$\frac{x_o}{2}$	$\frac{x_o}{\sqrt{2}}$	$\frac{\sqrt{3}x_o}{2}$	x_o
P.E	0	$\frac{E_o}{4} = 25\%$	$\frac{E_o}{2} = 50\%$	$\frac{3E_o}{4} = 75\%$	$E_o = 100\%$

Note:
P.E of horizontal mass-spring system is independent of mass of the body.

Kinetic Energy:

As $K.E = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{k}{m}}\sqrt{x_o^2 - x^2}$

$\Rightarrow K.E = \frac{1}{2}k(x_o^2 - x^2) \quad or$

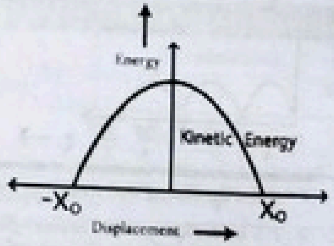
$K.E = \frac{1}{2}m\omega^2(x_o^2 - x^2)(\because k = m\omega^2)$

➢ K.E depends upon spring constant amplitude and displacement.

➢ K.E is independent of mass of the body.

➢ At mean position $x = 0 \Rightarrow$
$K.E = \frac{1}{2}kx_o^2 \ (maximum)$

➢ **At extreme position $x = x_o \Rightarrow$**
$K.E = 0 \ (minimum)$



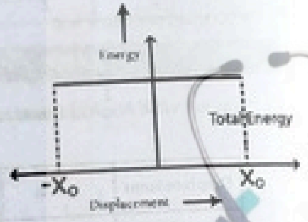
Displacement	$x = 0$	$x = \frac{x_o}{2}$	$x = \frac{x_o}{\sqrt{2}}$	$x = \frac{\sqrt{3}x_o}{2}$	x_o
P.E	0	$\frac{E_o}{4} = 25\%$	$\frac{E_o}{2} = 50\%$	$\frac{3E_o}{4} = 75\%$	$E_o = 100\%$
K.E	$E_o = 100\%$	$\frac{3E_o}{4} = 75\%$	$\frac{E_o}{2} = 50\%$	$\frac{E_o}{4} = 25\%$	0
T.E	E_o	E_o	E_o	E_o	E_o

Total Energy:

$$T.E = K.E + P.E$$

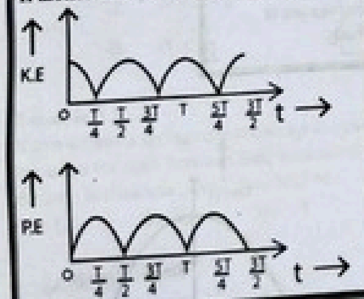
$$T.E = \frac{1}{2}k(x_o^2 - x^2) + \frac{1}{2}kx^2$$

$$T.E = \frac{1}{2}kx_o^2 \quad \text{or } T.E = \frac{1}{2}km\omega^2 x_o^2 (\because k = m\omega^2)$$

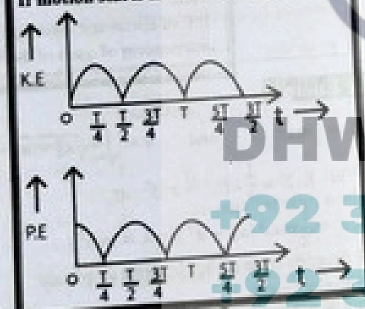


- T.E only depends upon spring constant and amplitude.
- T.E is directly proportional to square of amplitude ($T.E \propto x_o^2$)
- T.E of mass spring system is independent of displacement and mass of the body.

If motion starts from mean position



If motion starts from extreme position



- *K.E is oscillating with frequency frequency 2f and time period $\frac{T}{2}$*
- *P.E is oscillating with frequency frequency 2f and time period $\frac{T}{2}$*

Example:

At what displacement P.E and K.E of a body executing SHM are equal.

(a) $\frac{x_o}{2}$ (b) $\frac{x_o}{\sqrt{2}}$ (a) $\frac{\sqrt{3}x_o}{2}$ (d) x_o

Solution: P.E = K.E

$$\frac{1}{2}kx^2 = \frac{1}{2}k(x_o^2 - x^2)$$

$$x^2 = x_o^2 - x^2$$

$$2x^2 = x_o^2$$

$$x = \frac{x_o}{\sqrt{2}}$$

Quick Revision Chart:

t	0	$\frac{T}{12}$	$\frac{T}{8}$	$\frac{T}{6}$	$\frac{T}{4}$
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
x	0	$x = \frac{x_o}{2}$	$x = \frac{x_o}{\sqrt{2}}$	$x = \frac{\sqrt{3}x_o}{2}$	x_o
V	$v_o = x_o\omega$	$\frac{\sqrt{3}v_o}{2}$	$\frac{v_o}{\sqrt{2}}$	$\frac{v_o}{2}$	0
A	0	$\frac{a_o}{2}$	$\frac{a_o}{\sqrt{2}}$	$\frac{\sqrt{3}a_o}{2}$	$a_o = x_o\omega^2$
P.E	0	$\frac{E_o}{4}$	$\frac{E_o}{2}$	$\frac{3E_o}{4}$	$E_o = \frac{1}{2}kx_o^2$
K.E	0	$\frac{3E_o}{4}$	$\frac{E_o}{2}$	$\frac{E_o}{4}$	0

Quantity	At mean Position	At Extreme Position
Displacement	0	x_o (maximum)
Velocity	$x_o\omega$ (maximum)	0
Acceleration	0	$x_o\omega^2$ (maximum)
Force	0	$kx_o = mx_o\omega^2$ (maximum)
Momentum	$mx_o\omega$ (maximum)	0
P.E	0	$\frac{1}{2}kx_o^2$ (maximum)
K.E	$\frac{1}{2}kx_o^2$ (maximum)	0

WAVES

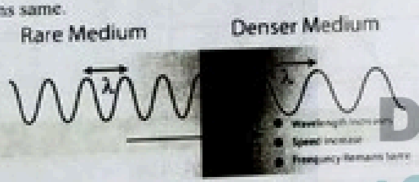
- ***Wave is disturbance in a medium which transfers energy and momentum from one region of space to another.***
- Wave do not transfers matter.

TYPES OF WAVES

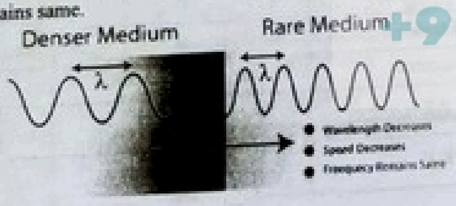
- Mechanical Waves
- Electromagnetic Waves
- Matter Waves

Mechanical Waves:

- ***They require medium for their propagation.***
- They are produced by vibrating matter particles.
- Water waves, waves in spring, waves in string, sound waves, ultrasound waves and infrasound waves are mechanical waves.
- In vacuum speed of mechanical waves is zero.
- When enter from rare to denser medium their speed and wavelength both increases but frequency remains same.



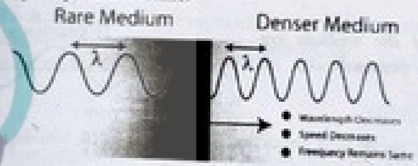
- When enter from denser to rare medium their speed and wavelength both decreases but frequency remains same.



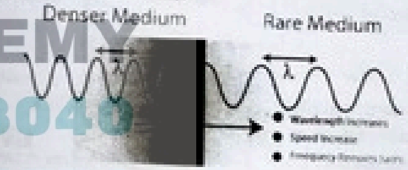
Wave:	Electromagnetic Waves:
Sound waves	20Hz—20000Hz
Infrasound	Less than 20Hz
Ultrasound	Greater than 20000Hz

Electromagnetic Waves:

- ***They do not require medium for their propagation.***
- Vibrating electric and magnetic field in perpendicular direction.
- Radio waves, micro waves, infrared, visible light, ultraviolet, x-rays and γ-rays are electromagnetic waves.
- Spectrum
- Visible light spectrum.
- In vacuum all electromagnetic waves have same $speed = 3 \times 10^8\ m/s$.
- Relation between wavelength and frequency is $f \propto \frac{1}{\lambda}$.
- When enter from rare to denser medium their speed and wavelength both decreases but frequency remains same.



- When enter from denser to rare medium their speed and wavelength both increases but frequency remains same.



In medium
$v \propto \lambda$

Example:
Which of following radiation will travel faster in glass
(A) Infrared (B) Visible light
(C) Ultraviolet (D) All have same speed

Answer: *Infrared*
Reason:
Infrared have greater wavelength than others so $v \propto \lambda$.

Matter Waves:

- ***Waves associated with moving particles.***
- Matter waves were proposed by De-Broglie.
- For example De-Broglie wavelength of a particle is given as

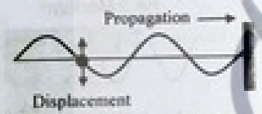
In terms of speed	In terms of momentum	In terms of energy	In terms of accelerating voltage
$\lambda = \frac{h}{mv}$	$\lambda = \frac{h}{p}$	$\lambda = \frac{h}{\sqrt{2mE}}$	$\lambda = \frac{h}{\sqrt{2meV}}$
$\lambda \propto \frac{1}{m}$ and $\lambda \propto \frac{1}{v}$	$\lambda \propto \frac{1}{p}$	$\lambda \propto \frac{1}{\sqrt{E}}$	$\lambda \propto \frac{1}{\sqrt{V}}$

PROGRESSIVE OR TRAVELLING WAVES

- ***Waves which transfer energy by moving away from source of disturbance are called progressive waves.***
- Transverse and longitudinal waves are two types of progressive waves.

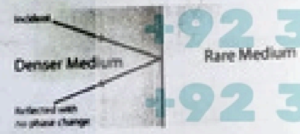
Transverse Waves:

Waves in which particles of the medium are displaced in a direction perpendicular to direction of propagation.



- Water waves, waves in string and all E.M waves (radio, microwaves, IR, visible, ultraviolet, x-rays, γ-rays) are transverse waves
  - When a transverse wave travelling in rare medium is reflected from denser medium it undergo a phase change of 180°.

  - When a transverse wave travelling in denser medium is reflected from rare medium no phase change occurs.

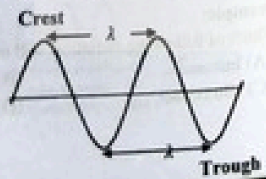


Crest:

Portion of wave in which particles are displaced above their mean position.

Trough:

Portion of wave in which particles are displaced below their mean position.



Wavelength:

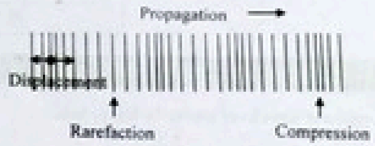
Distance between two consecutive crests or troughs is called wavelength.

- Transverse waves can be polarized.
- Transverse waves can be set up only in solids, in liquids and gases they are damped out quickly.

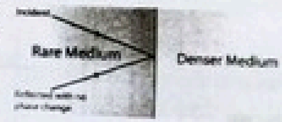
Longitudinal Waves:

Waves in which particles of the medium are displaced in a direction along the direction of propagation of waves.

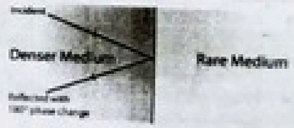
- Sound waves, ultrasound waves, infrasound waves and waves in spring are examples of longitudinal waves.



- ***Compression:*** Portion of the wave in which density of particles is high.
- ***Rarefaction:*** Portion of the wave in which density of particles is low.
  - When longitudinal waves travelling in rare medium is reflected from denser medium, no phase change occurs.

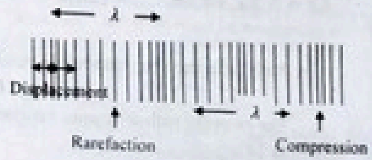


  - When longitudinal wave travelling in denser medium is reflected from rare medium it undergoes a phase change of 180°.



Wavelength:

Distance between two consecutive compressions or two consecutive rarefactions is called wavelength.



- Longitudinal waves can not be polarized.
- Longitudinal waves can be set up in all type of media solids, liquids and gases.

> **یاد رکھیں**
>
> Longitudinal اور Transverse ویوز میں فرق صرف Polarization کی base پر کر سکتے ہیں

PERIODIC WAVES:

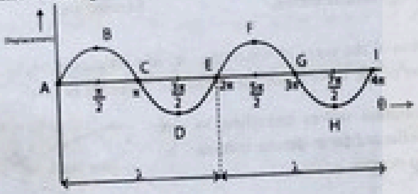
Continuous, regular and rhythmic disturbances in a medium are called periodic waves.

- Relation $v = f\lambda$ is only applicable for periodic waves.
  Speed: Distance covered by wave in one second.
  Wavelength: Distance between two consecutive points which are in phase.
  Frequency: Number of waves passing through a point in one second.

$$f = \frac{number\ of\ waves}{time}$$

PHASE RELATIONSHIP BETWEEN TWO POINTS ON A WAVE

Consider a waveform shown in figure below



In phase:

- Points having same displacements as well as same direction of motion are called in phase points.
- Phase difference between two in phase points is even π
  $\Delta\phi = 0, 2\pi, 4\pi, 6\pi, \ldots\ldots = (2n)\pi$

یاد رکھیں
Phase difference [illegible] Angle [illegible]
[illegible] angle [illegible] 2π [illegible]

- Path difference between two in phase points is integral multiple of λ

$$\Delta x = 0, \lambda, 2\lambda, 3\lambda, \ldots\ldots = n\lambda$$

For example path difference between A and E is λ and path difference between A and I is 2λ

Examples:

- Points A, E and I are in phase.
- Points B and F are in phase.
- Points D and H are in phase.
- Points C and G are in phase.

Out of phase:

- Points having opposite displacement or opposite direction of motion are called out of phase points.
- Phase difference between two out of phase points is odd π

$$\Delta\phi = 0, 2\pi, 4\pi, 6\pi, \ldots\ldots = (2n+1)\pi$$

Example: Phase difference between B and D $= \frac{3\pi}{2} - \frac{\pi}{2} = \pi$

- Path difference between two out of phase points is integral multiple of $\frac{\lambda}{2}$

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots = (2n+1)\frac{\lambda}{2}$$

For example path difference between A and C $= \frac{\lambda}{2}$.

Examples:

- Points A and C are out of phase.
- Points B and D are out of phase.

Relation between phase difference and path difference is given as

$$\Delta\phi = \frac{2\pi(\Delta x)}{\lambda}.$$

[illegible] $\Delta\phi$ [illegible] Δx [illegible]

Example:
Path difference between two points is $\frac{3\lambda}{4}$ then the points are:
(a) In phase (b) Out of Phase
(c) Coherent (d) Neither in phase nor out of phase

Answer:
neither in phase nor out of phase
Because Path difference is neither multiple of λ nor $\frac{\lambda}{2}$.

SPEED OF SOUND

- Sound waves (pressure waves) are longitudinal waves.
- In vacuum (free space) speed of sound is zero.
- In any medium speed of sound is given as

$$v = \sqrt{\frac{E}{\rho}}$$

Speed of sound only depends upon two factors

i. Modulus of elasticity of the medium or compressibility of medium.

$$v \propto \sqrt{E} \quad or \quad v \propto \frac{1}{\sqrt{compressibility}}$$

$$\left(\because E = \frac{1}{compressibility}\right)$$

ii. Density or inertia of the medium.

iii. Speed of sound is independent of frequency, wavelength or loudness.

iv. In solids molecules are closely spaced to each other as compared to liquids and gases that is why they respond to the disturbance more quickly as

OR $E_s > E_{liq} > E_g$ So $v_s > v_{liq} > v_g$

Example: Which of the following sound waves have greater speed in air
(a) 20 Hz (b) 10,000 Hz (c) 20,000 Hz (d) All have same ✓

SPEED OF SOUND IN AIR

Newton's Calculations:

For calculating speed of sound in air Newton assumed that when sound waves passes through air temperature of air remains constant (isothermal process).

- Boyle's law is applicable $(PV = constant)$.
- According to Newton or for isothermal process, modulus of elasticity of air is equal to pressure of air $(E = P = 1.01 \times 10^5 \text{pa})$
- According to Newton speed of sound in air is

$$\boxed{v = \sqrt{\frac{P}{\rho}}} \quad (\because E = P)$$

$$v = \sqrt{\frac{1.01 \times 10^5}{1.29}} = 280\ m/s$$

- Experimental value of speed of sound in air at standard temperature (0°C) is 332 m/s.
- There was about 16% error in Newton's calculations.

Laplace Correction:

- ***Laplace pointed out that compressions and rarefactions occurs so rapidly that heat produced during the compressions is confined to the region where it is generated and does not have time to flow to the cooler region where rarefaction occurs.***
- Temperature of air does not remain constant.
- Since no heat flow occurs so passage of sound is an adiabatic process.
- Relation between pressure and volume is $PV^\gamma = constant$.
- $\gamma = \frac{C_p}{C_v} = \frac{\text{molar specific heat at constant pressure}}{\text{molar specific heat at constant volume}}$
- γ has no unit, no dimensions and $\gamma > 1$.

Monoatomic gas	Diatomic gas	Polyatomic gas
$\gamma = 1.67$	$\gamma = 1.4$	$\gamma = 1.29$

- As air is almost diatomic so for air $\gamma = 1.4$.
- According to Laplace or for adiabatic process, modulus of elasticity of air is γ times pressure of air $(E = \gamma p)$
- According to Laplace, speed of sound in air is

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}} \quad (\because E = \gamma P)$$

$$v = \frac{1.4 \times 1.01 \times 10^5}{1.29} = 333\ m/s$$

Example:

If v_1 and v_2 are speeds of sound in air according to Newton and Laplace calculations then which of the following is true.

(a) $v_1 = v_2$ (b) $v_1 = \sqrt{\gamma} v_2$ (c) $v_2 = \sqrt{\gamma} v_1$ (d) $v_2 = \gamma v_1$

Solution:

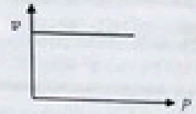
$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{P}{\rho}}}{\sqrt{\frac{\gamma P}{\rho}}} = \frac{1}{\sqrt{\gamma}}$$

$$\Rightarrow v_2 = \sqrt{\gamma} v_1$$

Effect of Pressure:

Since density is directly proportional the pressure $\left(\frac{P}{\rho} = constant\right)$ hence speed of sound is not effected by variation in pressure of air.

- Graph between speed of sound in air verses pressure is straight line



Example: If pressure of air is doubled then speed of sound in air will

(a) Become double (b) Become $\sqrt{2}$ times (c) Become half (d) Remain same✓

Effect of density:

- At constant pressure and temperature speed of sound in air is inversely proportional to root of density.

$$\boxed{v \propto \frac{1}{\sqrt{\rho}}} \quad OR \quad \frac{v_1}{v_2} = \frac{\sqrt{\rho_2}}{\sqrt{\rho_1}}$$

- Since ratio between densities of O_2 to H_2 is 16 : 1 hence ratio between speed of sound in O_2 to H_2 will be 1 : 4.
- $\frac{V_{O_2}}{V_{H_2}} = \frac{1}{4}$ OR $\frac{V_{H_2}}{V_{O_2}} = \frac{4}{1}$ OR $V_{H_2} = 4V_{O_2}$

Example:

At same temperature and pressure in which of following gas speed of sound will be minimum

(a) H_2✓ (b) O_2 (c) N_2 (d) None of these

Solution:

H_2 lowest density and

$$v \propto \frac{1}{\sqrt{\rho}}$$

Effect of Temperature:

- ***At constant pressure and temperature speed of sound increases by increasing temperature.***

  Reason: By increasing temperature, volume increases and density decreases so speed of sound increases.
- At any temperature t°C the volume of gas is given as $V_t = V_o(1 + \beta t)$

  Where β is coefficient of volume expansion and $\beta = \frac{1}{273}$

  $V_t = V_o\left(1 + \frac{t}{273}\right)$

- ***Speed of sound in air is directly proportional to square root of absolute temperature (Temperature in Kelvin)***

$$v \propto \sqrt{T} \quad \text{OR} \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \quad \text{OR} \quad T \propto v^2$$

Example:

At what temperature speed of sound will become double as that is at 10°C

(a) 40 °C (b) 313 °C
(c) 895 °C (d) 1132 °C

Solution: $v \propto \sqrt{T}$ or $T \propto v^2$

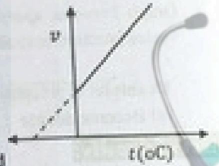
If v is doubled, Temperature should be 4 times

$$T = 4(10 + 273) = 4(283)$$
$$= 1132K = 1132 - 273 = 895°C$$

> **Note**
> If speed becomes n-times then absolute temperature will become n^2 times ($T' = n^2 T$)

- Speed of sound at any temperature t(°C) is given as

$$v_t = v_o + 0.61t \quad \text{if} \quad t \ll 273$$



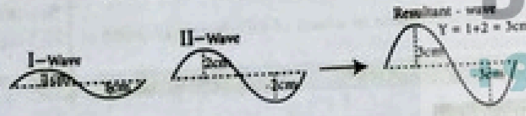
- With one degree or one Kelvin rise in temperature speed of sound increases by 0.61 m/s or 61 cm/s.

PRINCIPLE OF SUPERPOSITION

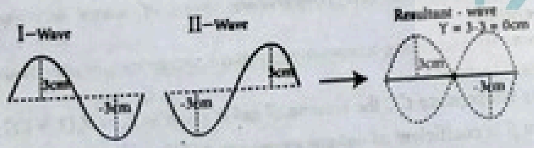
- If two or more waves are simultaneously acted on medium particle then resultant displacement of particle is algebraic sum of their individual displacements.

$$Y = Y_1 + Y_2 + \cdots \ldots + Y_n$$

Exmaple: 1



Exmaple: 2



Three cases of Principle of Superposition

- ***Interference:*** When two waves of same frequency and travelling in same direction superpose with each other.
- ***Beats:*** When two waves of slightly different frequencies but travelling in same direction superpose with each other.
- ***Stationary:*** When two waves of same frequency but travelling in opposite direction superpose with each other.

INTERFERENCE

- **Superposition of two waves of same frequency and travelling in same direction results a phenomenon called interference.**

Constructive Interference:

Constructive interference occurs if waves are in phase and they reinforce the effect of each other.

- Phase difference = even $\pi = (2n)\pi$
- Path difference = $n\lambda$ (integral multiple of λ).
- If A_1 and A_2 are amplitude of two waves then resultant amplitude will be $A_1 + A_2$.
- Relation between intensity and amplitude is $intensity \propto (amplitude)^2$

> **Note**
> Waves having constant phase difference are called coherent waves.

Destructive Interference:

Destructive interference occurs if waves are out of phase and they cancel out the effect of each other.

- Phase difference = odd $\pi = (2n + 1)\lambda$
- Path difference = $(2n + 1)\frac{\lambda}{2}$(odd integral multiple of $\frac{\lambda}{2}$).

Resultant amplitude will be $A_1 - A_2$.

> **یاد رکھیں**
> (2n + 1) ایک odd نمبر کو ظاہر کرتا ہے ...
> integer ... odd ...

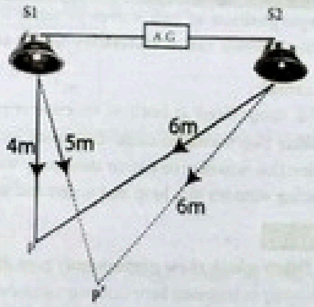
Example 1:

If two speakers are producing sound waves of wavelength 2m(λ = 2m) as shown in figure. If two waves have same amplitude then find resultant displacement at point P and P'.

At point P path difference between two waves is

$$\Delta S = 6m - 2m = 4m = 2(2m)$$
$$= 2\lambda \quad (\because \lambda = 2m)$$

Since path difference is 2λ hence 2nd order maxima occur at point P.



Resultant displacement = $A + A = 2A$

At point P' path difference between two waves is

$$\Delta S = 6m - 5m = 1m = \frac{\lambda}{2} (\because \lambda = 2m)$$

Since path difference is $\frac{\lambda}{2}$ hence 1st order maxima occur at point P.

Resultant displacement = $A - A = 0$

BEATS

- **Superposition of two waves having slightly different frequency but travelling in same direction results a phenomenon called beats.**
- Beats are periodic fluctuations between maximum and minimum sound.
- A single tuning fork produces sound waves of single frequency say $f = 32\ Hz$
- By loading some wax or plasticize or prongs of tuning forks its frequency decreases say $f = 30\ Hz$.
- If two tuning forks are sounded together beats are produced having beat frequency $2Hz = (32 - 30)$.
- Beat frequency is always equal to difference between the frequencies.

$$f_{beat} = f_A - f_B$$

Example: If two sound waves of frequencies 50 Hz are travelling in same direction then

$$f_{beat} = 54 - 50 = 4Hz$$

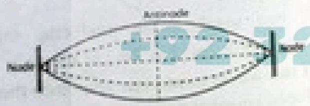
- If beat frequency (number of beats per second) is greater than 10Hz. Beats can not be distinguished by human ear.
- Beats are useful in tuning a string instruments.

STATIONARY WAVES

Superposition of two waves of same frequency but travelling in opposite direction results a phenomenon called stationary waves or standing waves.

Example:

If a string fixed at both of its ends is plucked from its center two waves of same frequency travels in opposite direction which give rise to stationary waves.



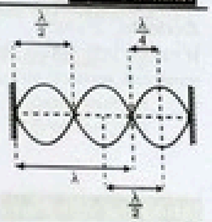
String vibrates in a loop and nodes and anti-nodes are formed.

Nodes:

Points which show permanently zero displacement are called nodes.

Energy is bounded between two consecutive nodes.

Anti-Nodes: ***Points which vibrating with maximum amplitude are called anti-nodes.***



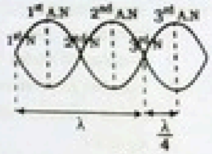
- All the particles of string execute SHM except nodes.
- Distance between two consecutive nodes is $\frac{\lambda}{2}$.
- Distance between node and next anti-node is $\frac{\lambda}{4}$.
- Distance between two consecutive antinodes is $\frac{\lambda}{2}$.

Example:

Consider string is vibrating in three loops. What is distance between first node and third anti-node.

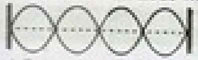
(a) $\frac{\lambda}{4}$ (b) $\frac{3\lambda}{2}$ (c) $\frac{3\lambda}{4}$ (d) $\frac{5\lambda}{4}$ ✓

$distance = \lambda + \frac{\lambda}{4} = \frac{5\lambda}{4}$



Example:

If 12cm string is vibrating in four segments (loops) then distance between node and next anti-node will be



(a) 0.75cm (b) 1.5cm (c) 3cm (d) 6cm

Solution: $L = 4\left(\frac{\lambda}{2}\right)$

$4\left(\frac{\lambda}{2}\right) = 12cm$

$\lambda = 6cm$ thus

$\left(\frac{\lambda}{4}\right) = 1.5cm$

STATIONARY WAVES IN STRETCHED STRING

- Waves travelling in stretched string are transverse waves.
- If a stretched string is plucked two transverse waves travelling in opposite direction give rise to stationary waves.

Speed of Waves: Speed of transverse waves in a stretched string is given as

$$v = \sqrt{\frac{F}{m}}$$

- ***F is tension in the string***
- ***m is mass per unit length of string***

Unit of m is kgm^{-1}

- Speed is directly proportional to square root of tension and inversely proportional to square root of mass per unit length of string.

$$v \propto \sqrt{F}$$

$$v \propto \frac{1}{\sqrt{m}}$$

- Speed of waves in stretched string is independent of length of string, number of loops and frequency of vibration.

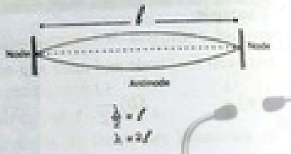
Example:
If tension in the string becomes four time than speed of transverse waves in string will become

(a) Four times (b) Two times
(c) One fourth (d) One half

Solution:
As $v \propto \sqrt{F}$
So, speed will become two times
[illegible] $\sqrt{\ }$ Tension [illegible]

I-mode of vibration:

If a string of length ℓ is plucked from length $\frac{\ell}{2}$ it will vibrate in single loop.



- Number of loops formed = 1
- Number of nodes formed = 2
- Number of Anti-nodes formed = 1
- String vibrates with maximum wavelength i.e. $\lambda_1 = 2\ell$
- String vibrates with minimum frequency i.e. $f_1 = \frac{1}{2\ell}\sqrt{\frac{F}{m}}$
- f_1 is known as fundamental frequency, fundamental tone or I-harmonic.

Law of length	*Law of tension*	*Law of mass*
$f \propto \frac{1}{\ell}$	$f \propto \sqrt{F}$	$f \propto \frac{1}{\sqrt{m}}$

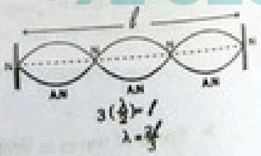
II mode of vibration:

If a string is plucked from length $\frac{\ell}{4}$ it vibrates in two loops.

- Number of loops formed = 2
- Number of nodes formed = 3
- Number of Anti-nodes formed = 2
- String vibrates with wavelength i.e. $\lambda_2 = \frac{\lambda_1}{2} = \ell$
- String vibrates with frequency $f_2 = 2f_1$

III mode of vibration:

If a string is plucked from length $\frac{\ell}{6}$ it vibrates in two loops.



- Number of loops formed = 3
- Number of nodes formed = 4
- Number of Anti-nodes formed = 3
- String vibrates with wavelength i.e. $\lambda_3 = \frac{\lambda_1}{3} = \frac{2\ell}{3}$
- String vibrates with frequency $f_2 = 2f_1$

Harmonics:

Such oscillations in which each frequency is integral multiple of fundamental frequency are called harmonics.

- Only harmonics are produced in stretched string having frequencies $f_1, 2f_1, 3f_1, 4f_1$ ..... *and* wavelength $\lambda_1, \frac{\lambda_1}{2}, \frac{\lambda_1}{3}, \frac{\lambda_1}{4}$ ......
- Frequencies other than harmonics are damped out quickly.

Example:
If fundamental frequency is 20Hz then which of the following frequency waves can not be produced in stretched string.

(a) 40 Hz (b) 60 Hz
(c) 70 Hz (d) 80 Hz

Solution:
70 Hz is not integral multiple of 20 Hz.

nth-Harmonic:

- String is plucked from = $\frac{\ell}{2n}$
- Number of loops formed = n
- Number of nodes formed = $n + 1$
- Number of antinodes formed = n

Frequency:

$f_n = nf_1$ OR $f_1 = \frac{1}{2\ell}\sqrt{\frac{F}{m}}$

Where n= 1,2,3,.........

Wavelength:

$\lambda_n = \frac{\lambda_1}{n}$

OR

$\lambda_n = \frac{2\ell}{n}$

Where n= 1,2,3,.........

Over Tones:

An overtone is any frequency among harmonic series that is greater than fundamental frequency.

Examples: Frequency of 1st overtone = $2f_1$
Frequency of 2nd overtone = $3f_1$
Frequency of 3rd overtone = $4f_1$

Frequency	f_1	$2f_1$	$3f_1$	$4f_1$	$5f_1$
Mode of vibration	First	Second	Third	Fourth	Fifth
Harmonic	First	Second	Third	Fourth	Fifth
Over tone	Fundamental tone	First	Second	Third	Fourth

APPLICATIONS

- Nuts on guitar are used to change the tension in string and thus change the frequency.
- String of different mass per unit length are used to produce note of different frequencies.
- One can move hand on the neck of guitar to change the length of string to produce note of different frequencies.

Example:
If frequency of 5[th] over tone is 60 Hz then frequency 2[nd] harmonic will be
(a) 10 HZ (b) 20 HZ (c) 25 HZ (d) 30 HZ

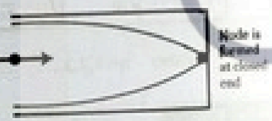
Solution:
5th over tone $= 6f_1 = 60$
$\Rightarrow f_1 = 10\ Hz$
Frequency 2nd harmonic $= 2f_1 = 20\ Hz$

Example:
If three consecutive frequencies of a harmonic series given as 60 Hz, 75 Hz, 90 Hz then frequency of 1[st] overtone will be
(a) 5 HZ (b) 15 HZ (c) 30 HZ (d) 60 HZ

Solution:
$f_1 = 75 - 60 = 15\ Hz$
Frequency of 1[st] over tone $= 2f_1 = 2(15) = 30\ Hz$

STATIONARY WAVES IN AIR COLUMN

- ***Longitudinal wave (sound or pressure waves) can produce stationary waves in air column.***
- When we blow air in an air column then the relation between incident and reflected wave depends on whether reflecting end is open or closed



- (i) At open end air molecules have complete freedom of motion and can vibrate with maximum displacement and thus it behave as anti node.
- (ii) At closed end motion of air molecules is restricted and displacement of air molecules remains permanently zero thus it behave as node.
- Although sound waves are longitudinal waves but the displacement of air molecules can be represented by a transverse wave.

Example: Which of the following mode of vibration can not be produced in air column?

(a) 
(b) 
(c) 
(d) All of these✓

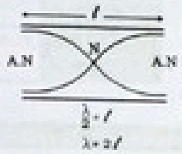
Reason: At open end always anti-node and at closed end always node is formed.

Case-I: Pipe is open at both ends

I-mode of vibration

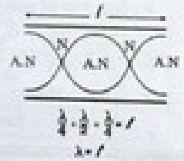
Consider a pipe of length ℓ open at both ends.

- Number of loops formed = 1
- Number of nodes formed = 1
- Number of anti-nodes formed = 2
- Wavelength: $\lambda = 2\ell$ (maximum wavelength)
- Frequency: $f_1 = \frac{v}{2\ell}$ (Where v is speed of sound in air)
- f_1 is known as fundamental frequency or fundamental harmonic or fundamental tone.



II-mode of vibration

- Number of loops formed = 2
- Number of nodes formed = 2
- Number of anti-nodes formed = 3
- Wavelength: $\lambda = \frac{\lambda_1}{2} \Rightarrow f_2 = \frac{2\ell}{2}$
- Frequency: $f_2 = 2f_1 \Rightarrow f_2 = \frac{2v}{2\ell}$



nth-Harmonic:

- Number of loops formed = n
- Number of nodes formed = n
- Number of antinodes formed = n+1

Frequency:

$$f_n = nf_1 \quad \text{OR} \quad f_n = \frac{nv}{2\ell}$$

Where n= 1,2,3,.........

Wavelength:

$$\lambda_n = \frac{\lambda_1}{n}$$

OR

$$\lambda_n = \frac{2\ell}{n}$$

Where n= 1,2, 3,.........

Harmonics:

Such oscillations in which each frequency is integral multiple of fundamental frequency are called harmonics.

- In an open end pipe only harmonic are produced having frequencies $f_1, 2f_1, 3f_1, 4f_1, \ldots\ldots$ and wave length $\lambda_1, \frac{\lambda_1}{2}, \frac{\lambda_1}{3}, \ldots\ldots$
- Frequencies other than harmonics are damped out quickly.

Over Tones:

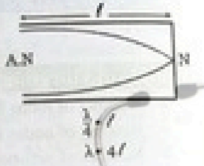
An overtone is any frequency among the harmonic that is greater than fundamental frequency.

Examples:
Frequency of 1[st] overtone $= 2f_1$
Frequency of 2[nd] overtone $= 3f_1$
Frequency of 3[rd] overtone $= 4f_1$

Frequency	f_1	$2f_1$	$3f_1$	$4f_1$	$5f_1$
Mode of vibration	First	Second	Third	Fourth	Fifth
Harmonic	First	Second	Third	Fourth	Fifth
Over tone	Fundamental tone	First	Second	Third	Fourth

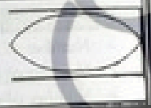
Case-II: Pipe is open at both ends

I-mode of vibration



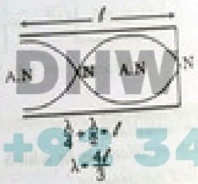
Consider a pipe of length ℓ open at both ends.

- Number of loops formed = $\frac{1}{2}$
- Number of nodes formed = 1
- Number of anti-nodes formed = 1
- Wavelength: $\lambda_1 = 4\ell$ (maximum)
- Frequency: $f_1 = \frac{v}{2\ell}$ (minimum)
- f_1 is known as fundamental frequency or fundamental harmonic or fundamental tone.



Note: Single loop is produced in closed end pipe as shown in the following figures
Because at open end always anti-nodes and at closed end always node is formed. Hence 2[nd] harmonic cannot be produced in closed end pipe.

II-mode of vibration



- Number of loops formed = $\frac{3}{2}$
- Number of nodes formed = 2
- Number of anti-nodes formed = 2
- Wavelength: $\lambda_3 = \frac{\lambda_1}{3} \Rightarrow \frac{4\ell}{3}$
- Frequency: $f_3 = 3f_1 \Rightarrow \frac{3v}{4\ell}$

nth-Harmonic:

- Number of loops formed = $\frac{n}{2}$
- Number of nodes formed = $\frac{n+1}{2}$
- Number of anti-nodes formed = $\frac{n+1}{2}$

Frequency:

$$f_n = nf_1 \qquad f_n = \frac{nv}{4\ell}$$

Where n= 1,3 ,5,.........

Wavelength:

$$\lambda_n = \frac{\lambda_1}{n} \quad \text{OR} \quad \lambda_n = \frac{4\ell}{n}$$

Where n= 1,3, 5,.........

Harmonics:

Such oscillations in which each frequency is integral multiple of fundamental frequency are called harmonics.

- If pipe is closed at one end only odd harmonics are produced having frequencies $f_1, 3f_1, 5f_1, \ldots\ldots$ and wavelength $\lambda_1, \frac{\lambda_1}{3}, \frac{\lambda_1}{5}, \ldots\ldots$
- Frequencies other than odd harmonics are damped out quickly.

Over Tones:

An overtone is any frequency among the harmonic series that is greater than fundamental frequency.

Examples: Frequency of 1[st] overtone = $3f_1$
Frequency of 2[nd] overtone = $5f_1$
Frequency of 3[rd] overtone = $7f_1$

Frequency	f_1	$3f_1$	$5f_1$	$7f_1$	$9f_1$
Mode of vibration	First	Second	Third	Fourth	Fifth
Harmonic	First	3[rd]	5[th]	7[th]	9[th]
Over tone	Fundamental tone	First	Second	Third	Fourth

Note: If pipe is open at both ends, both even and odd harmonics are produced. But if pipe is closed at one end only odd harmonics are produced. So open end pipe is richer in harmonics than closed end pipe.

DOPPLER'S EFFECT

Apparent change in frequency of waves due to relative motion between source and observer is called Doppler's effect.

- Doppler's effect was first observed for light coming from a distant star.
- Doppler's effect is applicable for all types of waves (longitudinal or Transverse, mechanical or electromagnetic)

Case I:

Frequency کی change صرف تب پیدا ہو گا جب source اور observer کی distance "change" ہو رہا ہو۔
(Doppler's effect is observed)

- If distance between source and observer decreases then frequency increases.
- If distance between source and observer increases then frequency decreases.
- If distance between source and observer does not change then frequency remains same and $\Delta f = 0$.

Case ii:

Relative speed صرف تب change ہوگی جب observer move کر رہا ہو۔

- If observer is moving towards the source then relative speed increases and
  $$v_{rel} = v + u_o$$
  where v is speed of the wave and u_o is speed of observer.
- If observer is moving away from source then relative speed of wave decreases and
  $$v_{rel} = v - u_o$$
- If observer is at rest then relative speed of wave does not change.

Case iii: (Doppler shift is produced only if source is moving)

Wave length میں change صرف تب پیدا ہوگا جب source move کرے گا۔

- If source is moving towards the observer than apparent wavelength decreases
  and $\lambda' = \lambda - \Delta\lambda$
- If source is moving away from the source apparent wavelength increases
  and $\lambda' = \lambda + \Delta\lambda$
- If source is at rest then wavelength remains same $\lambda' = \lambda$ and $\Delta\lambda = 0$

Doppler's Shift:

Change in wavelength $\Delta\lambda$ is known as Doppler shift and

$$\Delta\lambda = \frac{u_s}{f}$$

Doppler's shift صرف تب پیدا ہوگی جب source move کرے گا۔

Where u_s is speed of source and f is actual frequency

Doppler's shift only depends upon two factors.

(i) Speed of source $\Delta\lambda \propto u_s$

(ii) Actual Frequency $\Delta\lambda \propto \frac{1}{f}$

Example:

If two cars A and B horn the sound of same frequency while approaching an observer with velocities 20 m/s and 30 m/s respectively then Doppler shift is maximum for?

(a) Car A (b)Car B (c) Same for A and B (d) Zero for A and B

Solution:
Car A
Reason:
$(\Delta\lambda \propto u_s)$

Example:

If a star is moving towards the earth with speed v then Doppler shift is maximum for?

(a) IR✓ (b) uv (c) Visible light (d) same for all E.M waves

Solution:
IR
Reason:
$\left(\Delta\lambda \propto \frac{1}{f}\right)$

Apparent Frequency:

General relation for apparent frequency is given as

$$f' = \frac{v \pm u_o}{v \mp u_s} \times f$$

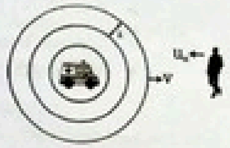
اگر observer دور جا رہا ہو تو -ve لگائیں۔

اگر observer قریب آ رہا ہو تو +ve لگائیں۔

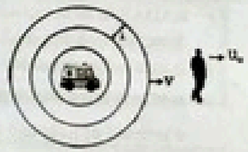
اگر source قریب آ رہا ہو تو +ve لگائیں۔

اگر source دور جا رہا ہو تو +ve لگائیں۔

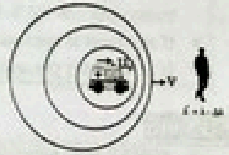
1. **If observer is moving towards stationary source** $(u_s = 0)$
   - $f' = \left(\frac{v+u_o}{v}\right)f$
   - Apparent frequency increases.
   - Pitch of sound increases.
   - Wavelength remains same and $\Delta\lambda = 0$



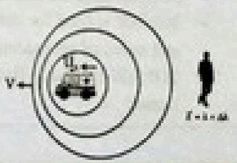
2. **If observer is moving away from stationary source** $(u_s = 0)$
   - $f' = \left(\frac{v-u_o}{v}\right)f$
   - Apparent frequency decreases.
   - Pitch of sound decreases.
   - Wavelength remains same and $\Delta\lambda = 0$



3. **If source is moving towards stationary source** $(u_o = 0)$
   - $f' = \left(\frac{v}{v-u_s}\right)f$
   - Apparent frequency increases.
   - Pitch of sound increases.
   - Wavelength remains same and $\Delta\lambda = \frac{u_s}{f}$

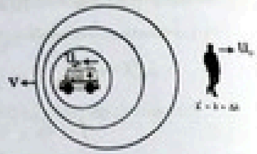


4. **If source is moving away from stationary source** $(u_o = 0)$
   - $f' = \left(\frac{v}{v+u_s}\right)f$
   - Apparent frequency decreases($f' < f$).
   - Pitch of sound increases.



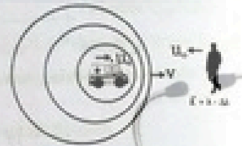
- Wavelength increases ($\lambda' > \lambda$).

5. If source and observer are moving away from each other.



- $f' = \left(\frac{v+u_o}{v-u_s}\right) f$
- Apparent frequency increases ($f' > f$).
- Wavelength decreases ($\lambda' < \lambda$)

6. If source and observer are moving towards each other.



- $f' = \left(\frac{v-u_o}{v+u_s}\right) f$
- Apparent frequency decreases ($f' < f$).
- Wavelength increases $\lambda' > \lambda$

APPLICATIONS OF DOPPLER'S EFFECTS

RADAR:

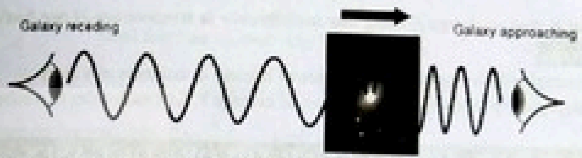
- RADAR is an abbreviation of " radio amplification detection and ranging".
- It uses radio waves for detection of objects.
- RADAR is used to determine the range $\left(R = \frac{1}{2}ct\right)$ where c is speed of light and t is time between transmission and reception of radio signal.
- RADAR is used to determine speed of objects ($u_s = f(\Delta\lambda)$)where f is actual frequency of transmitter and $\Delta\lambda$ is Doppler's shift.
- If an object is approaching the RADAR then λ decreases.
- If an object is preceding the RADAR then λ increases.

SONAR:

- SONAR is an abbreviation of "sound navigation and ranging".
- SONAR uses ultrasound waves.
- SONAR is used for detection and to determine range and speed of submarines.
- SONAR is used to fixed sea depth and undersea mines.

Red Shift:
If a star is moving away from earth its spectrum is shifted towards longer wavelength (towards red) it is known as red shift.

Blue Shift:
If star is moving away from earth its spectrum is shifted towards shorter wavelength (blue end) it is known as blue shift.



Light waves are stretched (red-shift) Light waves are compressed (blue-shift)

RADAR Speed Trap:

- RADAR speed trap uses microwaves to determine vehicles speed.
- Speed of vehicle can be calculated by ($u_s = f\Delta\lambda$) where f is actual frequency and $\Delta\lambda$ is Doppler shift.

Do You Know
Ultrasound waves of frequencies 5 MHz to 10 MHz are directed towards artery to monitor blood flow through major arteries apparent frequency depends upon velocity of flow of blood

Do You Know
Echolocation allows dolphins to detect small difference in the shape, size and thickness of object.

Do You Know
Bat navigates and find food by echo location

UNIT 05 HEAT AND THERMODYNAMICS

Heat: ***Heat is type of energy flowing due to difference in temperature of two bodies.***

Temperature:

- Macroscopically temperature is measure of hotness or coldness of a body.
- Microscopically temperature is measure of average K.E of molecules of a substance.

KINETIC MOLECULAR THEORY OF GASES

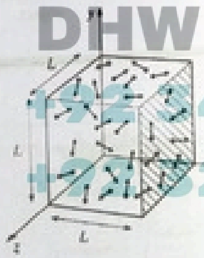
- There are two evidence for kinetic molecular theory of gases.
  (i) Diffusion (ii) Brownian motion

Postulates:

- All gases consists of very small discrete particles called molecules.
- Gas molecules are in state of random motion in all possible directions with different velocities.
- Molecules of gas are constantly colliding with each other and with walls of container.
- Collision of molecules is perfectly elastic collision.
- Molecules of gas do not exert force on each other except collision.
- Volume occupied by gas molecules is negligible as compared to volume of gas (volume of container).

PRESSURE OF GAS

Pressure of gas is defined as momentum transferred to walls of container per second per unit area due to continuous collisions of gas molecules with the walls of container.



- Consider N number of molecules enclosed in a cubical box of side 'ℓ'.
- If a molecule of mass 'm' moving with velocity v_{1x} rebounds back with same velocity (due to elastic collision) then

$$\text{Change in momentum} = P_f - P_i = (-mv_1) - (mv_{1x})$$
$$= -2mv_1x$$

- *Time between two consecutive collisions* $\Delta t = \frac{2\ell}{v_{1x}}$.
- No. of collisions per second $= \frac{v_{1x}}{2\ell}$
- *Force exerted by molecule on the wall* $= \frac{\text{change in momentum}}{\text{time}} = \frac{2mv_{1x}}{\frac{2\ell}{v_{1x}}}$

$$\boxed{F_{1x} = \frac{mv_{1x}^2}{\ell}}$$

$$\text{Total force on the wall ABCD} = F_{1x} + F_{2x} + \cdots \ldots + F_{Nx}$$
$$= \frac{mv_{1x}^2}{\ell} + \frac{mv_{2x}^2}{\ell} + \cdots + \frac{mv_{Nx}^2}{\ell}$$

- Pressure on the wall ABCD $= \frac{m}{\ell^3}(v_{1x}^2 + v_{2x}^2 + \cdots + v_{Nx}^2)$

$$P_x = \frac{m\ell}{\ell^3}\left(\frac{v_{1x}^2 + v_{2x}^2 + \cdots + v_{Nx}^2}{N}\right)$$

$$\boxed{P_x = \rho < v_x^2 >}$$

- ***Density of gas*** $= \frac{\text{Total mass}}{\text{Total volume}} = \rho = \frac{mN}{\ell^3}$
- ***Average velocity*** $= \frac{v_1+v_2+\cdots+v_N}{N} = < v > = 0$
- ***Average square velocity*** $= \frac{v_1^2+v_2^2+\cdots+v_N^2}{N} = < v^2 > \neq 0$
- ***Root mean square velocity*** $= \sqrt{< v^2 >} = \sqrt{\frac{v_1^2+v_2^2+\cdots+v_N^2}{N}} \neq 0$
- Since molecules are in random motion hence $< v_x^2 > = < v_y^2 > = < v_z^2 >$
- Similarly $P_y = \rho < v_y^2 >$ and $P_z = \rho < v_z^2 >$
- $< v^2 > = < v_x^2 > + < v_y^2 > + < v_z^2 > = 3 < v_x^2 >$

$$\Rightarrow < v_x^2 > = \frac{1}{3} < v^2 > \quad and\ P = \frac{1}{3}\rho < v^2 >$$

- $P_x = P_y = P_z$ (Pascal's law)
- Net pressure on any wall of container is given as

$$\boxed{P = \rho < v^2 >}$$

Example: If velocity or speed of each molecule is doubled then pressure of gas will become
(a) Double (b) Half (c) Four Times ✓ (d) Remains same

Solution: As $P \propto < v^2 >$
If v is doubled, P will become four times.

Example:
If average square velocity of gas molecules is doubled, pressure of gas will become
(a) Double ✓ (b) Half (c) Four Times (d) Remains same

Solution: As $P \propto \langle v^2 \rangle$
If $\langle v^2 \rangle$ is doubled, P will become double.

Kinetic Equation of gas

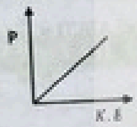
- Kinetic equation of gas is given as $P = \frac{1}{3}\rho \langle v^2 \rangle = \frac{mN}{3V}\langle v^2 \rangle$

$$\Rightarrow PV = \frac{1}{3} mN \langle v^2 \rangle$$

Relation between Pressure and <K.E>

- $P = \frac{mN}{3V}\langle v^2 \rangle = \frac{2N}{3V}\langle \frac{1}{2}mv^2 \rangle$

$$\Rightarrow \boxed{P = \frac{2N}{3V}\langle K.E \rangle}$$



- If $\frac{N}{V} = N_o = constant$ then $P \propto \langle K.E \rangle$

General Gas Equation:

- General gas equation for 'n' moles is given as $\boxed{PV = nRT}$

Where R is general gas constant and in SI units $R = 8.314\ J mol^{-1} K^{-1}$

- If n and T are constant then $P \propto \frac{1}{V}$ *(Boyle's law)*
- If n and T are constant then $V \propto T$ *(Charle's law)*
- If P and T are constant then $V \propto n$ *(Avogadro's law)*
- If n and V are constant then $P \propto T$ *(Lussac's law)*

Boltzman Constant: (k)

The ratio of general gas constant to Avogadro's number is called Boltzmann constant.

- $\boxed{k = \frac{R}{N_A}}$ and $k = \frac{8.314}{6.02 \times 10^{23}} = 1.38 \times 10^{-23} J K^{-1}$

Example: The product of Boltzmann and Avogadro's number (in SI units) is always equal to
(a) 8.314 ✓ (b) 6.02×10^{23} (c) 1.38×10^{-23} (d) one

Solution:
$N_A \times k = R = 8.314$

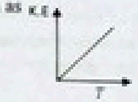
Example: The ratio of unit of Boltzmann to unit of General gas constant is equal to
(a) mole (b) $mole^{-1}$ ✓ (c) $J\ mole^{-1}$ (d) one

Solution:
$\frac{J\ mole^{-1} K^{-1}}{J K^{-1}} = mole^{-1}$

Relation between Temperature and <K.E>

- At any temperature T (*Kelvin*) <K.E> is given as

$$\boxed{\langle K.E \rangle = \frac{3}{2}kT}$$



- $\langle K.E \rangle \propto T$
- ***Average K.E of molecules only depends upon temperature and independent of nature of gas.***

Example: At room temperature which of the following gas molecules have greater average kinetic energy.
(a) H_2 (b) CO_2 (c) N_2 (d) All have same energy ✓

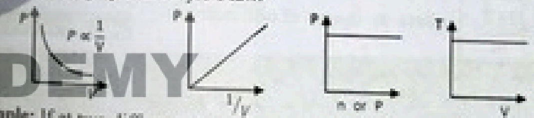
Solution:
Since temperature is same so <K.E> is also same.

Example:
At temperature 27°C, average K.E of gas molecules will be
(a) $6 \times 10^{-21} J$ (b) $60 \times 10^{-21} J$
(c) $600 \times 10^{-21} J$ (d) $0.6 \times 10^{-21} J$

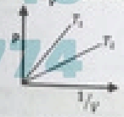
Solution: $\langle K.E \rangle = \frac{3}{2}kT$
(put $T = 27 + 273 = 300K$)
$= \frac{3}{2}(1.38 \times 10^{-23})(300)$
(put $3 \times 1.38 = 4$)
$= 600 \times 10^{-23} J = 6 \times 10^{-21} J$

Boyle's Law:

- ***If number of moles of gas and temperature are kept constant then volume of gas is inversely proportional to pressure.***
- $V \propto \frac{1}{P}$ (*If n and T = constant*)
- $PV = k$ (*where k is constant and k = nRT*)
- $P_1V_1 = P_2V_2$
- Different graph for Boyle's law:



Example: If at two different constant temperatures T_1 and T_2 the graph between P and $\frac{1}{V}$ is shown in the figure then.



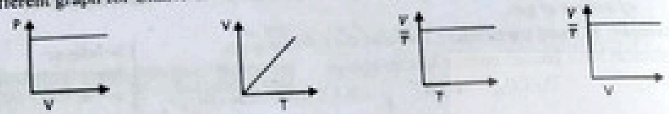
(a) $T_1 > T_2$ (b) $T_1 < T_2$
(c) $T_1 = T_2$ (d) None

Solution:
$$Slope\ of\ graph = \frac{P}{\left(\frac{1}{V}\right)}$$
$= PV = k = nRT$
$\Rightarrow Slope \propto T$

Slope of T_2 larger then T_1
so $T_2 > T_1$

Charle's Law

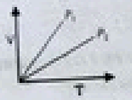
- ***If number of moles of a gas and pressure are kept constant then volume of gas is directly proportional to its absolute temperature.***
- $V \propto T$ (If n and P are constant)
- $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ or $V_1T_2 = V_2T_1$
- Different graph for Charle's law.



- $\frac{V}{T} = k$ (where k is constant and $k = \frac{nR}{P}$) as $PV = nRT \Rightarrow \frac{V}{T} = \frac{nR}{P}$

Example: If at two different constant pressures P_1 and P_2 the graph between volume and temperature of a gas is shown in the figure below then

(a) $P_1 > P_2$ (b) $P_1 < P_2$
(c) $P_1 = P_2$ (d) None of these



Solution:
Slope of graph $= \frac{V}{T} = \frac{nR}{P}$
$\Rightarrow Slope \propto \frac{1}{P}$
$so\ P_2 > P_1$

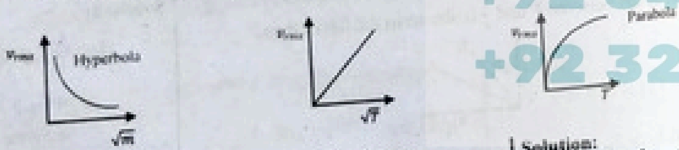
Root mean square velocity:

- Root mean square velocity is given as $v_{rms} = \frac{v_1^2 + v_2^2 + \cdots + v_n^2}{N}$

$$v_{rms} = \sqrt{\frac{3KT}{m}}$$

where 'm' is mass of each molecule.

- Root mean square velocity depends upon
  1. Temperature $(v \propto \sqrt{T})$
  2. Nature of the gas $\left(v \propto \frac{1}{\sqrt{m}}\right)$



Example: At room temperature which of the following gas molecules will have greater root mean square velocity
(a) H_2 ✓ (b) CO_2 (c) N_2 (d) All have same energy

Solution:
Since $v_{rms} \propto \frac{1}{\sqrt{m}}$ Hence H_2 has greater v_{rms}.

Other relations for root mean square velocity are

$v_{rms} = \sqrt{\frac{3RT}{M}}$	$v_{rms} = \sqrt{\frac{3PV}{M}}$	$v_{rms} = \sqrt{\frac{3P}{\rho}}$

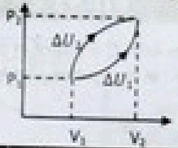
Internal Energy:

Sum of all forms of molecule energies (P.E and K.E) is called internal energy.

- For ideal gas molecule $P.E = 0$, because there is no force of attraction or repulsion between molecules.
- If gas is mono-atomic molecules posses only translational K.E.
- If gas is diatomic or polyatomic molecules posses translational, vibrational and rotational K.E.
- Since K.E only depends upon temperature and internal energy of an ideal gas only depends upon temperature.
- By increases temperature internal energy increases and vice versa.
- At constant temperature (isothermal process) $U = constant$ and $\Delta U = 0$.
- ***A function which only depends upon initial and final states and independent of path followed is called state function.***
- Change in internal energy is a **state function (i.e independent of path followed)**
  - If a system changes state (P_1, V_1) to (P_2, V_2) along two different path as shown in figure then

$$\Delta U_1 = \Delta U_2$$

یاد رکھیں
Ideal گیس کے لیے Internal Energy صرف گیس کے Temperature پر depend کرتی ہے
$Internal\ Energy \propto Temperature$



Sign convensions:

Heat added to system	Heat removed from system	Work done by the system	Work done on the system	If internal energy increases then ΔU	If internal energy decreases then ΔU
+ve	−ve	+ve	−ve	+ve	−ve

Work done in thermodynamics:

- Work done at constant pressure P is given as

$$W = P\Delta V \text{ or } W = P(V_f - V_i)$$

- Work done by the system is taken positive.
- Work done on the system is taken negative.
- ***Area under P − V graph is equal to work done.***

Example: If volume V of gas increased by 200% at pressure P then the work done by the system is

(a) PV (b) $2PV$
(c) $3PV$ (d) $4PV$

Solution: $V_f = V + \frac{200V}{100} = 3V$

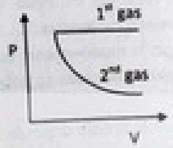
$$W = P(V_f - V_i)$$
$$= P(3V - V) = 2PV$$

Example: P-V graph for two gases is shown in the figure below. For same change in volume which of following option is correct.

(a) $W_1 = W_2$ (b) $W_1 < W_2$
(c) $W_1 > W_2$ (d) $W_2 > W_1$ and $W_1 = 0$

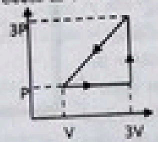
[illegible] area [illegible] case [illegible] work [illegible]

Solution: $W_1 > W_2$



Example: A system undergoes a cycle process as shown in the figure below then work done will be

(a) PV (b) $2PV$
(c) $3PV$ (d) $4PV$



Solution:
$W_1 = Area\ of\ triangle$
$$= \frac{1}{2}(3P - P)(3V - V)$$
$$= \frac{1}{2}(2P)(2V) = 2PV$$

FIRST LAW OF THERMODYNAMICS

When heat Q is added to system it appears as increases in internal energy which is stored in the system plus work done by the system.

$$Q = \Delta U + W$$

Where ΔU is change in internal energy($\Delta U = U_f - U_i$)

- First law of thermodynamics is actually law of conservation of energy in thermodynamics

Example 1: Bicycle Pump:

When we compress the air in a bicycle pump by closing its nozzle. Mechanical work is done on it. Since no heat flow occurs so this mechanical energy is converted into internal energy. Thus internal energy increases and its temperature increases.

Example 2: Metabolism:

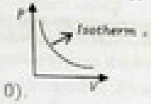
Energy transforming process that occurs in an organism is called metabolism.

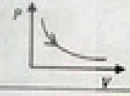
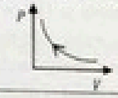
- Energy from food we eat is stored in the body in form of internal energy.
- By doing some mechanical work internal energy decreases. $\Delta U = Q - W$

Isothermal Process:

A process which is carried out at constant temperature is called isothermal process.

- Since $T = constant$, hence internal energy also remains constant and $\Delta U = 0$.
- Boyle's law is valid $PV = constant$
- Curve representing the isothermal process is called isotherm.
- First law of thermodynamics takes the form $Q = W(\because \Delta U = 0)$.
- Mostly process which are carried out slowly are isothermal.
- For isothermal process modulus of elasticity of gas is equal to pressure of the gas ($E = P$).

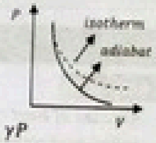


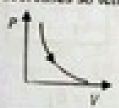
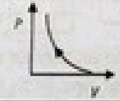
Isothermal Expansion	Isothermal Compression
➢ $T = Constant$ and $\Delta U = 0$ ➢ Work done is +ve. ➢ Mechanical energy is converted into heat. $Q = W$ ([illegible] mechanical energy [illegible] heat [illegible])	➢ $\Delta U = 0$ and $T = Constant$ ➢ Work done is −ve. ➢ Heat is converted into mechanical energy. $-Q = -W$ ([illegible] heat [illegible] work [illegible])
	

Adiabatic Process

A process in which no heat enters or leaves from the system $Q = 0$.

- Temperature of the system may increase or decrease.
- Relation between pressure and volume is $PV^\gamma = constant$
- Curve representing the adiabatic process is called adiabat.
- Adiabat is γ times steeper than isotherm.
- First law of thermodynamics takes form $W = -\Delta U(\because Q = 0)$
- Mostly process which are carried out rapidly are adiabatic.
- For adiabatic process modulus of elasticity of gas is given as $E = \gamma P$



Adiabatic Expansion	Adiabatic Compression
➢ $Q = 0$ ➢ Work done is +ve. ➢ Internal energy is converted into work. ➢ $W = -\Delta U$ ([illegible] work [illegible] internal energy [illegible]) ➢ Internal energy decreases so temperature decreases.	➢ $Q = 0$ ➢ Work done is −ve. ➢ Mechanical energy is converted into internal energy $\Delta U = -W$ ([illegible] work [illegible] internal energy [illegible]) ➢ Internal energy increases so temperature increases.
	

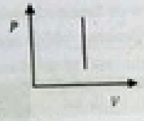
Examples:

i. Compressions and rarefactions of air through which sound waves are passing.
ii. Rapid escape of air from burst tyre.
iii. Cloud formation.

Isochoric process (Isometric process):

A process which is carried out at constant volume is called isochoric process ($\Delta V = 0$).

- No work is done ($W = P\Delta V = 0$).
- First law takes the form. $Q = \Delta U$
- If heat is added to system, internal energy increases thus temperature increases.
- If heat is removed from system, internal energy decreases.

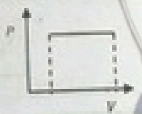


Example: Pressure cooker is an example of isochoric process in which volume is kept constant so $W = 0$ and heat added is entirely converted into internal energy.

Isobaric Process:

A process carried out at constant pressure is called isobaric process.

- Work is given as ($W = P\Delta V$).
- In isobaric expansion heat is partially converted into internal energy and partially into work.
- In isobaric compression work done on the system is partially converted into internal energy and partially into work.



Isothermal	Adiabatic	Isochoric
$\Delta U = 0$	$Q = 0$	$W = 0$
$Q = W$	$W = -\Delta U$	$Q = \Delta U$
$E = P$	$E = \gamma P$	$E = \infty$

MOLAR SPECIFIC HEAT

Heat Capacity

Amount of heat required to raise the temperature of a substance through one Kelvin is called heat capacity.

- It is denoted by C and $C = \frac{Q}{\Delta T}$
- Its SI unit is JK^{-1} and in terms of base unit is $Kgm^2s^{-2}K^{-1}$.
- Boltzmann constant, entropy and heat capacity have same units.
- Heat capacity depends upon two factors
  (i) Nature of substance (ii) Amount of substance.

$C \propto m$ or $\frac{C_1}{C_2} = \frac{m_1}{m_2}$ (m is mass) and $C \propto n$ or $\frac{C_1}{C_2} = \frac{n_1}{n_2}$ (n is number of moles)

- Heat capacity for adiabatic process is zero. ($\because Q = 0$).
- Heat capacity for isothermal process is infinite. ($\because \Delta T = 0$)

Specific Heat:

Amount of heat required to raise the temperature of 1 kg substance through one Kelvin is called specific heat.

- It is denoted by 'C' and $C = \frac{Q}{m\Delta T}$
- Its SI unit is $JKg^{-1}K^{-1}$ and in terms of base units is $m^2s^{-2}K^{-1}$.
- **Specific heat only depends upon nature of substance and independent of amount of substance.**
- *For adiabatic process* $Q = 0 \Rightarrow c = \frac{Q}{m\Delta T} = 0$
- *For isothermal process* $\Delta T = 0 \Rightarrow c = \frac{Q}{m\Delta T} = \infty$

Example: Ratio of specific heat of 2kg water to specific heat of 4kg is
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

Solution: 1 : 1
(specific heat is independent of amount of substance)

Molar Specific Heat:

Amount of heat required to raise the temperature of one mole substance through one Kelvin is called molar specific heat

- It is denoted by C and $C = \frac{Q}{n\Delta T}$
- Its SI unit is $Jmol^{-1}K^{-1}$
- General gas constant and molar specific heat have same units.
- Molar specific heat only depends upon nature of substance and independent of amount of substance.
- *For adiabatic process* $Q = 0 \Rightarrow C = \frac{Q}{n\Delta T} = 0$
- *For isothermal process* $\Delta T = 0 \Rightarrow C = \frac{Q}{n\Delta T} = \infty$

In case of solids or liquids

- When solids or liquids are heated their volume approximately remains constant.

$$V \approx constant$$

$$\Rightarrow \Delta V = 0 \text{ and } W = 0$$

$$\Rightarrow Q = \Delta U$$

- Heat is entirely converted into internal energy and no heat is used in doing work.

$Q_v = nC_v\Delta T$ And $\Delta U = nC_v\Delta T$

In case of gases

- For gases, there are two types of molar specific heat
  1. Molar specific heat at constant volume
  2. Molar specific heat at constant pressure.

Molar specific heat at constant volume	Molar specific heat at constant Pressure
➢ $V = constant \Rightarrow \Delta V = 0 \Rightarrow W = 0$ ➢ First law takes the form. $Q = \Delta U$ ➢ Heat is entirely converted into internal energy and no heat is used in doing work $\Delta U = nC_v\Delta T$	➢ $W = P\Delta V$ ➢ First law take the form $Q_P = \Delta U + P\Delta V$ ➢ Heat is partially converted into internal energy and partially used in doing work.

- ***Molar specific heat at constant pressure is always greater than molar specific heat at constant volume*** ($C_P > C_V$)
- $(C_P - C_V = R)$
- $\gamma = \frac{\text{molar specific heat at constant pressure}}{\text{molar specific heat at constant volume}} \Rightarrow$ $\boxed{\gamma = \frac{C_P}{C_V}}$
- The value of γ is always greater than 1 and it has no dimension and no unit.

Gas	Mono Atomic	Diatomic	Polyatomic
$\gamma = \frac{C_P}{C_V}$	$\frac{5}{3} = 1.29$	$\frac{7}{5} = 1.4$	$\frac{9}{7} = 1.29$
$C_V = \frac{R}{\gamma - 1}$	$\frac{3R}{2}$	$\frac{5R}{2}$	$\frac{7R}{2}$
$C_P = \frac{\gamma R}{\gamma - 1}$	$\frac{5R}{2}$	$\frac{7R}{2}$	$\frac{9R}{2}$

[illegible] C_V [illegible] $\frac{R}{2}$ [illegible] multiply [illegible]

[illegible] C_P [illegible] $\frac{R}{2}$ [illegible] multiply [illegible]

Example:
In case of Helium Gas the value of molar specific heat at constant volume is

(a) $\frac{3R}{2}$ ✓ (b) $\frac{5R}{2}$ (c) $\frac{7R}{2}$ (d) $\frac{9R}{2}$

Solution:
Helium is a mono atomic gas

UNIT 06 ⟫ ELECTOSTATICS

ELECTRIC CHARGE:

Charge is property associated with matter due to which it produces and experiences electric and magnetic effects.

- There are two types of charge (i) Positive charge (ii) Negative charge
- Like charges repel each other and unlike charges attract each other.

Example: Which of the following is greatest value of charge?

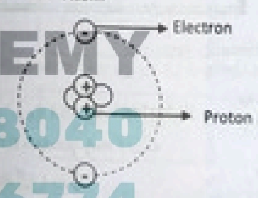
(a) 10 C (b) 20 C (c) -5 C (d) -50 C✓

- The smallest value of charge which can exist independently is $e = 1.6 \times 10^{-19}C$

Particle	Electron or $-ve\ \beta$	Proton or $+ve\ \beta$	Neutron or γ-rays	α-particle
Charge	$-e = -1.6 \times 10^{-19}C$	$+e = 1.6 \times 10^{-19}C$	Zero	$+2e = 3.2 \times 10^{-19}C$

- SI unit of charge is coulomb (in terms of base units A.s)
- Every atom is electrically neutral (No. of electrons = No. of Protons)

Example: He-Atom

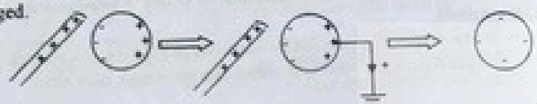


Net charge = $+2e - 2e = 0$

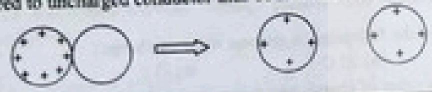
- *Charge is quantized* (charge is always an integral multiple of 'e') $\boxed{Q = ne}$
- **Law of conservation of charge**
  Charge neither be created and nor be destroyed but can be transferred from one body to another and total charge always remains constant.
- Charge given to a conductor always resides on the outer surface.

Methods of charging:

1. ***By Friction:*** By rubbing two bodies together both bodies are equally and oppositely charged due to transfer of electrons from one body to another.
   Examples: **(i)** When glass rod is rubbed with the silk, the glass rod becomes positively charged and silk is negatively charged. **(ii)** Clouds also get charged by friction.
2. ***By Electrostatic Induction:*** If a charged body is brought near a neutral body one side of neutral body (closer to charged body) is oppositely charged and while the other side is similarly charged.



3. ***By Conduction:*** When a charged conductor in contact with an uncharged conductor, some charge is transferred to uncharged conductor thus both conductors are similarly charged.



COULOMB'S LAW

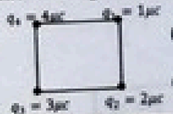
Force between two point charges is directly proportional to product of magnitude of charges and inversely proportional to square of the distance between them

Note
Coulomb's Law is applicable only for point charge.

$F \propto q_1 q_2$ and $F \propto \frac{1}{r^2}$

$\Rightarrow$ $F = \frac{Kq_1q_2}{r^2}$ Where k is proportionality constant

Example: If charges are placed at corners of a square as shown in figure below then force is maximum between



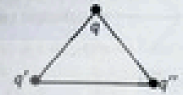
(a) q_1 and q_2 (b) q_2 and q_3
(c) q_3 and q_4 (d) q_1 and q_4

Solution: q_3 and q_4
جن charges کا product زیادہ ہوگا ان کے درمیان force زیادہ ہوگی

Constant	Value in SI-units	SI unit	In terms of base units	Dimensions
K (in free space)	9×10^9	Nm^2C^{-2}	$Kgm^3s^{-4}A^{-2}$	$ML^3T^{-4}A^{-2}$
ϵ_o	8.85×10^{-12}	$C^2N^{-1}m^{-2}$	$Kg^{-1}m^{-3}s^4A^2$	$M^{-1}L^{-3}s^4A^{-2}$

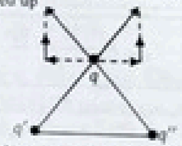
- ϵ_o is permittivity of free space and $K = \frac{1}{4\pi\epsilon_o}$
- $F = \frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r^2}$

Examples: Three equal and similar charges are placed at the corners of an equilateral triangle as shown in the figure then resultant force on q is acting along



(A) towards right (B) towards left
(C) upward (D) downward

Solution Horizontal component are cancelled and vertical component are added up



So the resultant force on q is in upward direction

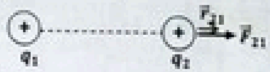
اس طرح کے questions کو solve کرنے کے لیے پہلے forces کو ان کے components میں resolve کریں پھر دیکھیں کون سے components add ہو رہے ہیں اور کون سے cancel ہو رہے ہیں۔

1. Force ہمیشہ charges کو ملانے والی line کے along ہوگی۔
2. components لیتے وقت جس angle کے سامنے والا component ہو اس کے ساتھ sine اور دوسرے کے ساتھ Cos ہوگا

Vector Form of Coulomb's Law:

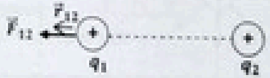
- Force exerted by q_1 on q_2 is given

$\vec{F}_{21} = \frac{Kq_1q_2}{r^2}\hat{r}_{21}$



- Force exerted by q_2 on q_1 is given as

$\vec{F}_{12} = \frac{Kq_1q_2}{r^2}\hat{r}_{12}$



- Both forces are always equal in magnitude but opposite in direction.
- *Coulomb's law obey the Newton's 3rd law of motion.*

$\vec{F}_{12} = -\vec{F}_{21}$

Example: If 4C and 6C charges are placed near each other the ratio of forces acting on the charges will be
(a)2 : 3 (b)3 : 2 (c)4 : 1 (d)1 : 1✓

دو charges ایک دوسرے پر برابر force لگاتے ہیں چاہے charge کی مقدار کچھ بھی ہو

Effect of medium:

- **Presence of dielectric medium (insulator) always reduces the electric force between the charges by a factor ϵ_r.**

$$F_{med} = \frac{F_{vac}}{\epsilon_r}$$ ([illegible] Quantities [illegible] relation [illegible])

or $$F_{med} = \frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q_1q_2}{r^2}$$

ϵ_r is known as → Relative permittivity / → Dielectric constant / → Dielectric coefficient

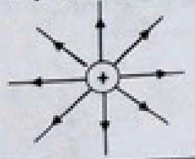
- ϵ_r is constant for given material and different for different materials.
- Value of ϵ_r is always greater than or equal to one for vaccume: $\epsilon_r = 1$, for insulating materials $\epsilon_r > 1$, For metals: $\epsilon_r = \infty$
- ϵ_r has no units, no dimensions.

Material	ϵ_r	Material	ϵ_r
Air	1.0006	Ammonia Liquid	22-25
Bakelite	5-18	Germanium	16
Glass	4.8-10	Paraffined paper	2
Rubber	2.94	Teflon	
Transformer oil	2.1	Water	78.5

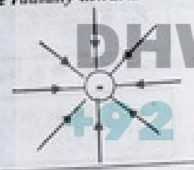
Electric Field Lines

- ***Lines which provide information about strength and direction of electric field are called electric field lines.***
- The concept of field lines was introduced by Michael Faraday.

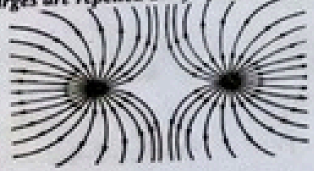
(i) Electric field lines due to positive point charge are radially outward.



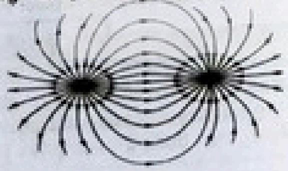
(ii) Electric field lines due to negative point charge are radially inward.



(iii) Electric field lines due to two similar charges are repelled away by charges.



(iv) Electric field lines due to two opposite charges are pulled towards charges.

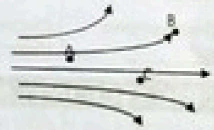


Properties:

i. Electric field lines are imaginary lines to visualize the electric field.
ii. No. of electric field lines ∝ magnitude charge.
iii. Electric field lines originate from positive charges and ends on negative charges.
iv. Lines are closer where field is strong and lines are path farther apart where field is weak.
v. Tangent to field lines at any point gives the direction of electric field at that point.
vi. No two field lines can cross each other because electric field has only one direction at a given point.
vii. If the distance between the lines remains same then field is uniform and if the distance between the line does not remain same then the field is non uniform.

Example: A non-uniform electric field is shown in the figure then at which of the following point electric field is maximum

(a) A ✓ (b) B (c) C (d) same at all points



Electric field:

The space around the charge in which its electric force acts on other charge is called electric field

ELECTRIC FIELD STRENGTH/INTENSITY

Electric field intensity at any point is defined as electric force per unit charge placed at that point

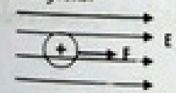
$$\vec{E} = \frac{\vec{F}}{q}$$

- Electric field intensity is a vector quantity.
- SI unit $= NC^{-1} = Vm^{-1}$, unit in terms of base units $= kgms^{-3}A^{-1}$.
- Dimensions $= [MLT^{-3}A^{-1}]$

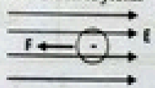
Electric force: Force acting on a charge 'q' in an electric field $\vec{E}$ is $\vec{F_e} = q\vec{E}$

- Electric force on a charge only depends upon magnitude of charge and electric field.
- Electric force is independent of mass, velocity or direction of motion of charged particle.

Electric force on positive charge is always parallel to electric field.

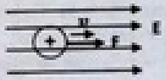
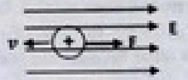
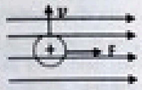


Electric force on negative charge is always anti-parallel to electric field.

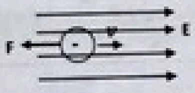
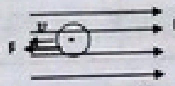
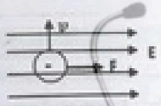


➢ ***Electric force can accelerate, decelerate and deflect the charge particle.***

In case of positive charge

(a) ***When $\vec{v}$ is parallel to $\vec{E}$***	(b) ***When $\vec{v}$ is anti-parallel to $\vec{E}$***	(c) ***When $\vec{v}$ is perpendicular to $\vec{E}$***
		
Electric field accelerate the charge	Electric field decelerate the charge	Electric field only deflect the charge

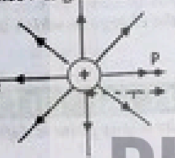
In case of negative charge.

(a) ***When $\vec{v}$ is parallel to $\vec{E}$***	(b) ***When $\vec{v}$ is anti-parallel to $\vec{E}$***	(c) ***When $\vec{v}$ is perpendicular to $\vec{E}$***
		
Electric field decelerate the charge	Electric field accelerate the charge	Electric field only deflect the charge

ELECTRIC FIEL INTENSITY DUE TO POINT CHARGE

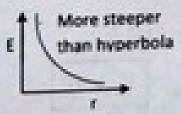
Electric field intensity due to point charge at any distance r is given as

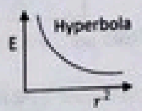
$$E = \frac{Kq}{r^2}$$

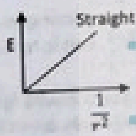


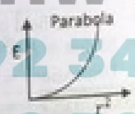
It only depends upon charge, distance from the charge And nature of the dielectric medium

$E \propto q$ OR $E \propto \frac{1}{r^2}$









Effect of Medium:

Presence of dielectric medium always reduces the electric field intensity by factor ϵ_r times.

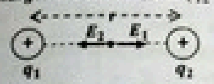
$$E_{med} = \frac{E_{vac}}{\epsilon_r}$$

(اگر دو Quantities معلوم ہوں تو relation سے تیسری کو معلوم کریں)

$$E_{med} = \frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q}{r^2}$$

Zero Field Location:

When charges are similar, Let $(q_2 < q_1)$



جب چارج similar ہوں تو zero field چارجز کے between ہوگا

اور smaller charge کے قریب ہوگا

If $\vec{E} = \vec{E}_1 - \vec{E}_2 = 0$ then $E_1 = E_2$

$$\Rightarrow \frac{kq_1}{r_1^2} = \frac{kq_2}{r_2^2} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{q_1}{q_2}}$$

$$\Rightarrow r_1 = \frac{r}{\sqrt{\frac{q_2}{q_1}}+1} \text{ and } r_2 = \frac{r}{\sqrt{\frac{q_1}{q_2}}+1}$$

Where r_1 and r_2 are distances of zero field location from the charges q_1 and q_2 respectively

When charges are opposite, Let $(q_2 < q_1)$

جب چارج opposite ہوں تو zero field چارجز کے باہر کسی ایک side پر

اور smaller چارج کے قریب ہوگا

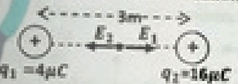
If $\vec{E} = \vec{E}_1 - \vec{E}_2 = 0$ then $E_1 = E_2$

$$\Rightarrow \frac{kq_1}{r_1^2} = \frac{kq_2}{r_2^2} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{q_1}{q_2}}$$

$$\Rightarrow r_1 = \frac{r}{\sqrt{\frac{q_2}{q_1}}-1} \text{ and } r_2 = \frac{r}{\sqrt{\frac{q_1}{q_2}}-1}$$

Where r_1 and r_2 are distances of zero field location from the charges q_1 and q_2 respectively

Examples: If $4\mu C$ and $16\mu C$ charges are separated by distance 3m. Then zero field location lies



(a) 1m from 16μC
(b) 2m from 4μC
(c) 1m from 4μC✓
(d) 4m from 4μC

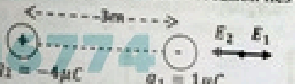
Solution:

As $r_1 = \frac{r}{\sqrt{\frac{q_2}{q_1}}+1}$

$$r_1 = \frac{3}{\sqrt{\frac{16}{4}}+1} = \frac{3}{2+1} = 1m$$

$$r_2 = 3 - 1 = 2m$$

Examples: If $1\mu C$ and $-4\mu C$ charges are separated by distance 3m. Then zero field location lies



(a) 1m from -4μC
(b) 3m from -4μC
(c) 3m from 1μC✓
(d) 6m from 1μC

Solution:

As $r_1 = \frac{r}{\sqrt{\frac{q_2}{q_1}}-1}$

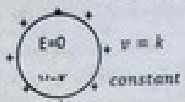
$$r_1 = \frac{3}{\sqrt{\frac{4}{1}}-1} = \frac{3}{2-1} = 3m$$

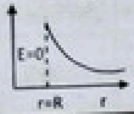
$$r_2 = 3 + 3 = 6m$$

ELECTRIC FIEL INTENSITY FOR DIFFERENT CHARGE DISTRIBUTION

(i) ***Charge conducting sphere (or shell of charge)***

- Electric field inside the hollow charged sphere is zero.
- Outside the sphere hollow charged sphere behave like a point charge.
- Electric potential inside the sphere is constant.





اس region کی electric field زیرو ہے
اس region کی potential مستقل ہے

Inside the sphere or shell ($r < R$)	On surface of sphere or shell ($r = R$)	On surface of sphere or shell ($r > R$)
$E = 0$	$E = \frac{kq}{R^2}$	$E = \frac{kq}{r^2}$

> To eliminate the stray of Electric field interference sensitive electronic devices such as T.V or computer are often enclosed in metal boxes

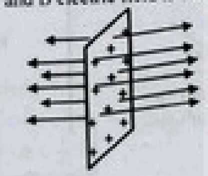
(ii) *Infinite sheet of charge:*

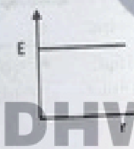
➢ Electric field intensity due to infinite sheet of charge is independent of distance from the sheet. For example at point A and B electric field is same.

$$E = \frac{\sigma}{2\varepsilon_o}$$

OR

$$E = \frac{Q}{2A\varepsilon_o}$$





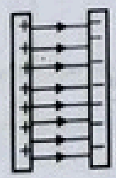
(ii) ***Between two closely spaced and oppositely charged plates** (capacitor):*

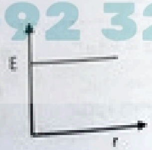
➢ Electric field intensity between two oppositely charged plates is also independent of distance.

$$E = \frac{\sigma}{\varepsilon_o}$$

OR

$$E = \frac{Q}{A\varepsilon_o}$$





Electric Potential: (Absolute Potential)

Electric potential at any point is defined as work done in bringing a unit positive charge from infinity to that point while keeping the charge in equilibrium.

$$V = \frac{W}{q_o}$$

➢ Potential is scalar quantity.
➢ SI unit = volt = $JC^{-1} = kgm^2s^{-3}A^{-1}$, Dimensions = $[ML^2T^{-3}A^{-1}]$

Potential difference:

Potential difference between two points is defined as work done in bringing a unit positive charge from one point to another while keeping the charge in equilibrium.

$$V_A - V_B = \frac{W}{q_o} \qquad \Delta V = \frac{W}{q_o}$$

Work done on the charge:

When a particle of charge q and mass m passes through P.d V then work done on it is given as

$$W = qV$$

➢ $W \propto q$ and $W \propto V$
➢ *W is independent of mass of particle*

Change in potential energy:

When a particle of charge q and mass m passes through P.d. V then change in P.E is given as

$$\Delta P.E = qV$$

➢ $\Delta P.E \propto q$ and $\Delta P.E \propto V$
➢ $\Delta P.E$ is independent of mass of particle

Gain in speed:

The gain in speed of particle when it is accelerated by P.d V is given as

$$v = \sqrt{\frac{2qV}{m}}$$

➢ $v \propto \sqrt{q}$
➢ $v \propto \sqrt{V}$
➢ $v \propto \frac{1}{\sqrt{m}}$

Gain in momentum

The gain in momentum of a particle when it is accelerated through P.d V is given as

$$p = \sqrt{2mqV}$$

➢ $p \propto \sqrt{m}$
➢ $p \propto \sqrt{q}$
➢ $p \propto \sqrt{V}$

Change in kinetic energy:

When a particle of charge q and mass m passes through P.d. V then change in K.E is given as

$$\Delta K.E$$

➢ $\Delta K.E \propto q$ and $\Delta K.E \propto V$
➢ $\Delta K.E$ is independent of mass of particle

Example:

If an electron and a proton are accelerated through same P.d then which of the following statement is true

(a) both will gain equal K.E
(b) electron will gain greater speed
(c) proton will gain greater momentum
(d) all of these ✓

Example:
If an electron and a proton are accelerated through same P.d then which one will gain greater momentum
(a) Electron (b) Proton (c) Both gain same (d) None

Solution: As $m_p > m_e$
and $P \propto \sqrt{m}$
So proton will gain greater momentum.

Potential Gradient

- **The quantity $\left(\frac{\Delta V}{\Delta r}\right)$ which gives the maximum rate of change of potential Δr with distance is called potential gradient.**
- Its SI unit is $Vm^{-1} = NC^{-1} = kgms^{-3}A^{-1}$
- Electric field intensity and potential gradient have same units.
- Potential gradient is a vector quantity.

- If electric field in a given region is zero then electric potential in this region is constant
- If electric field is constant it means that electric potential is changing at constant rate in this region

Relation with electric field

- Electric field is equal to – ve potential gradient.

$$\vec{E} = -\frac{\Delta V}{\Delta r}$$

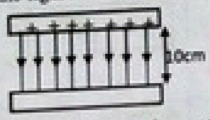
- If $E = 0 \Rightarrow \Delta V = 0$ and $V = constant$
- If $V = constant$ then $\Delta V = 0 \Rightarrow E = 0$
- In direction of electric field potential decreases.
- In opposite direction of electric field potential increases.
- In perpendicular direction of electric field potential remains same.

If V is P.d between two points separated by distance d then magnitude of electric field between the points is

$$\vec{E} = \frac{V}{d}$$

(اگر E,V اور d میں سے کوئی دو quantities معلوم ہوں
تو اس relation سے تیسری کو معلوم کریں)

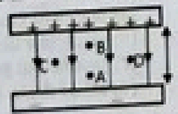
Example: P.d. between two oppositely charged parallel plates is 12V as shown in the figure then electric field between the plates will be



(a) $1.2NC^{-1}$ (b) $12NC^{-1}$ (c) $120NC^{-1}$ (d) Zero

Solution:
$$E = \frac{V}{d}$$
$$= \frac{12}{10 \times 10^{-2}}$$
$$= 120NC^{-1}$$

Example: P.d. between two oppositely charged parallel plates is 12V as shown in the figure then electric field between the plates will be



(a) Potential is maximum at B (b) Potential is minimum at A
(c) Potential is same at C & D (d) All of these✓

Solution:
Correct option is d
In direction of electric field potential decreases but in perpendicular direction potential remains constant.

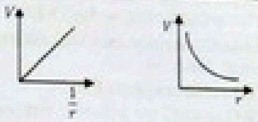
ELECTRIC POTENTIAL DUE TO POINT CHARGE

Electric potential at any distance r due to a point charge q is given as

$$V = \frac{kq}{r}$$

$$V \propto q \quad \text{and} \quad V \propto \frac{1}{r}$$

Note
i. Potential due to +ve charge is +ve
ii. Potential due to –ve charge is –ve

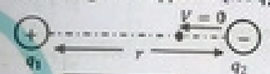


- In presence of dielectric medium potential decreases by ϵ_r times

$$V_{med} = \frac{V_{vac}}{\epsilon_r} \quad OR \quad V_{med} = \frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q}{r}$$

Zero Potential Location

a) When charges are opposite ($let\ q_2 < q_1$)



$$\frac{r_1}{r_2} = \frac{q_1}{q_2}$$

یاد رکھیے
اگر چارج opposite ہوں تو Zero Potential
چارج کے درمیان نہیں ہوگا smaller چارج کے قریب ہوگا

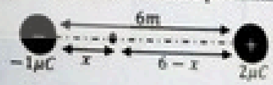
Distance of zero potential point from first charge	Distance of zero potential point from second charge
$r_1 = \frac{q_1 r}{q_1 + q_2}$	$r_2 = \frac{q_2 r}{q_1 + q_2}$

Example: Potential at a point midway between two equal and opposite charges is

(a) $\frac{kq}{r}$ (b) $\frac{2kq}{r}$ (b) $\frac{kq}{2r}$ (d) zero

Solution:
$$V = V_+ + V_-$$
$$= \frac{kq}{r} - \frac{kq}{r}$$
$$= 0$$

Example: If $-1\mu C$ and $2\mu C$ charges are separated by distance 6m then find the position of point where potential is zero



(a) 2m from $-1\mu C$ (b)2m from $2\mu C$
(c)1m from $-1\mu C$ (d)1m from $2\mu C$

Solution:
$$x = \frac{q_1 r}{q_1 + q_2}$$
$$x = \frac{1 \times 6}{1 + 2} = 2m$$
zero potential point lies 2m from $-1\mu C$ charge and 4m from $2\mu C$ charge

b) ***When charges are similar (both are +ve or both are −ve) then zero potential point lies at infinity.***

Electro Cardio Graphy:(ECG)

An ECG records the voltages between points on human skin generated by heart and it provide information about the performance of heart.

Electroencephelography:(EEG)

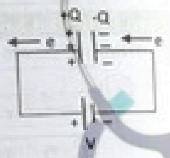
An EEG records the potential difference created by brain and provide information about for abnormal behavior.

Electroretinoraphy:(ERG)

An ERG records the potential difference generated by retina

CAPACITOR

Capacitor is a device which can store electric charge.

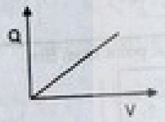


- When battery is connected across the plates of parallel plate capacitor, battery removes electrons from one plate and transfers electrons to other plate.
- Capacitor continue charging until its P.d becomes equal to P.d of the battery.
- Net charge on a capacitor is always zero.

Charge on the capacitor is directly proportional to P.d across the plates of capacitor.

$$Q \propto V \quad OR \quad Q = CV$$

Where C is proportionality constant known as capacitance.



- Slope $= \frac{Q}{V} = C$
- Area under the graph = P.E $= \frac{1}{2}QV$

Electric field intensity between plates of capacitor is

$$E = \frac{\sigma}{\epsilon_o \epsilon_r} = \frac{Q}{A\epsilon_o \epsilon_r}$$

Relation between electric field and P.d across the capacitor is

$$E = \frac{V}{d}$$

Electric force between the plates of capacitor is given as

$$F = QE$$

$$F = Q\left(\frac{Q}{2A\epsilon_o \epsilon_r}\right)$$

$$F = \frac{Q^2}{2A\epsilon_o \epsilon_r}$$

CAPACITANCE

Capacitance of parallel plate capacitor is defined as amount of charge on one plate necessary to raise its potential by one volt w.r.t. other plate.

$$C = \frac{Q}{V}$$

Note

C is proportionality constant and it is independent of Q and V

- SI unit of capacitance is farad.
- $F = CV^{-1} = kg^{-1}m^{-2}s^4A^{-2}$
  (Dimensions $= [M^{-1}L^{-2}T^4A^{-2}]$)
- Capacitance of parallel plate capacitor is given as

$$C_{med} = \frac{A\epsilon_o \epsilon_r}{d} \quad OR \quad C_{vac} = \frac{A\epsilon_o}{d}$$

It only depends upon three factors.

(i). Area of the plates $C \propto A$.

(ii) Distance between the plates $C \propto \frac{1}{d}$

(iii) Medium between the plates $C \propto \epsilon_r$

یاد رکھیں

Capacitor کی plates کے درمیان dielectric رکھنے سے capacitance ϵ_r گنا بڑھ جاتی ہے

Examples:

If length, width and thickness of plates of a capacitor are doubled then its capacitance will become

(a) Double (b) Four Times
(c) Eight Times (d) Remains same

Solution Since C is independent of thickness and $C \propto A \Rightarrow C \propto length \times width$
Hence capacitance becomes four times.

Dielectric Constant:

The ratio of capacitance of parallel plate capacitor when dielectric is as medium between the plates to capacitance of parallel plate capacitor when vacuum is medium between the plates.

$$\epsilon_r = \frac{C_{med}}{C_{vac}}$$

- It only depends on nature of the medium
- Its value is always greater or equal to one
- It has no unit no dimensions

Since $C_{med} > C_{vac} \Rightarrow \epsilon_r > 1$

Energy Stored in Capacitor:

- ***Capacitor is a device which can store the charge, alternatively capacitor is a device which can store electrical energy.***
- Charge stored on plates of capacitor posses electrical P.E arises due to work done by battery to deposite charge on the plates.
- P.E stored on the plates of capacitor is given as

i. $P.E = \frac{1}{2}QV$

ii. $P.E = \frac{1}{2}CV^2$

iii. $P.E = \frac{Q^2}{2C}$

> اگر اننا relation کب use کرنا ہے question میں دی گئی information پر depend کرتا ہے

- Energy stored in the capacitor can be regarded that energy is stored in form of electric field between the plates instead of P.E of the charges on the plates .

$P.E = \frac{1}{2}\epsilon_o\epsilon_r E^2(Ad)$ And $Energy\ density = \frac{1}{2}\epsilon_o\epsilon_r E^2$

Three Important cases for capacitor

1. ***If battery remains connected across capacitor or capacitors are connected in parallel then $V = constant$***

As $Q = CV \Rightarrow Q \propto C$ or $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$

and $P.E = \frac{1}{2}CV^2 \Rightarrow P.E \propto C$ or $\frac{P.E_1}{P.E_2} = \frac{C_1}{C_2}$

Example:
A capacitor is connected across a 12V battery. If a dielectric medium is introduced between the plates then which of the following statement is true.
(a) Capacitance increases (b)Charge increases
(c) Energy increases (d) All of these✓

Solution:
As $V = constant$ so $Q \propto C$ and $P.E \propto C$
By introducing dielectric C increases so Q and P.E also increases.

Example:
If $4\mu F$ and $6\mu F$ capacitor are connected in parallel that the ratio between charge stored in capacitor and energy stored in capacitor will be
(a) $2:3$ and $2:3$✓ (b) $2:3$ and $3:2$
(c) $3:2$ and $3:2$ (d) $3:2$ and $2:3$

Solution
:Since $V = constant$.
Hence $\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{4}{6} = 2:3$
and $\frac{P.E_1}{P.E_2} = \frac{C_1}{C_2} = \frac{4}{6} = 2:3$

2. ***If battery is not connected to capacitor (isolated capacitor) or capacitor are connected in series then $Q = constant$***

$Q = CV \Rightarrow V \propto \frac{1}{C}$ and $\frac{V_1}{V_2} = \frac{C_2}{C_1}$

and $P.E = \frac{Q^2}{2C} \Rightarrow P.E \propto \frac{1}{C}$ and $\frac{P.E_1}{P.E_2} = \frac{C_2}{C_1}$

Example: If distance between the plates of a charged capacitor is doubled then energy stored in capacitor will become
(a) Double✓ (b) Half
(c) Four Times (d) Remains same

Solution
As $Q = constant$
so $P.E \propto \frac{1}{C}$ and $C \propto \frac{1}{d}$
by doubling distance C becomes half thus energy becomes double

Example: If $C_1 = 2\mu F$ and $C_2 = 4\mu F$ are connected in series then ratio between their voltages will be
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 4

Solution: As $Q = constant$
so $V \propto \frac{1}{C}$
As $\frac{C_1}{C_2} = \frac{1}{2} \Rightarrow \frac{V_1}{V_2} = \frac{2}{1}$

3. ***If area of plates distance between the plates and medium between the plates are not changed then $C = constant$***

As $Q = CV \Rightarrow Q \propto V$ or $\frac{Q_1}{Q_2} = \frac{V_1}{V_2}$

and $P.E = \frac{1}{2}CV^2 \Rightarrow P.E \propto V^2$

Example: If voltage across capacitor is doubled then energy stored in capacitor will becomes
(a) Double (b) Half
(c) Four Times✓ (d) Remains same

Solution: As $C = constant$
$P.E = \frac{1}{2}CV^2 \Rightarrow P.E \propto V^2$
If V is doubled then energy will become four times

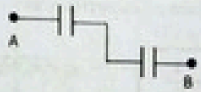
SERIES COMBINATION OF CAPACITORS

If capacitors end to end such that same charge is stored across all of them then this combination is known as series combination.

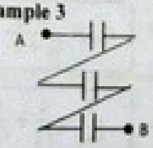
(اگر capacitors ایک ہی path میں connected ہوں تو capacitors سیریز میں ہوں گے)

Example 1

Example 2



Example 3



➢ When capacitors are connected in series charge on each capacitor is same

$$Q_1 = Q_2 = Q \text{ (Total charge)}$$

$$\Rightarrow \frac{Q_1}{Q_2} = 1$$

$$Q_1 = Q_2 = C_{eq}V$$

(کسی بھی capacitor پر چارج معلوم کرنے کے لیے پہلے C_{eq} معلوم کریں)

When capacitors are connected in series, total voltage is divided among the capacitors.

$$V = V_1 + V_2$$

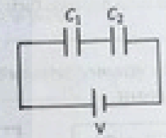
As Q is same so $V \propto \frac{1}{C}$ and $\frac{V_1}{V_2} = \frac{C_2}{C_1}$

(جس کی capacitance زیادہ ہو گی اس کے across کم Drop ہو گا)

Voltage Divider Rule:

If two capacitors are connected in series with voltage V as shown in figure

$$\frac{V_1}{V_2} = \frac{C_2}{C_1} \quad \text{and} \quad \frac{V_1}{V_2} = \frac{C_2}{C_1}$$



Voltage drop across the capacitor C_1 and C_2 is given as

$$V_1 = \frac{C_2}{C_1 + C_2}V \qquad V_2 = \frac{C_1}{C_1 + C_2}V$$

یاد رکھیں

(اگر capacitors کی capacitances برابر ہوں تو ان میں voltage کی drop برابر ہو گا)

➢ As Q is same so $P.E = \frac{Q^2}{2C} \Rightarrow P.E \propto \frac{1}{C}$ (جس کی capacitance زیادہ ہو گی اس میں کم انرجی store ہو گی)

Equivalent Capacitance:

If 'n' number of capacitors are connected in series

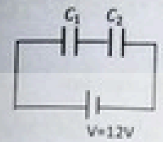
1. $C_{eq} = \frac{C}{n}$ (سب سے پہلے دیکھیں اگر capacitors کی values same ہیں تو capacitance کو تعداد سے divide کر دیں)

2. $C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{\text{Product of capacitances}}{\text{Sum of capacitances}}$ (اگر دو capacitor کی different values ہوں تو اس فارمولے کو use کریں)

3. $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$ (اگر دو سے زیادہ different values کے capacitor ہوں تو اس فارمولے کو use کریں)

4. $C_{eq} < C_{min}$
5. To decrease the capacitance capacitors are connected in series. (C_{eq} کی value سب سے چھوٹی capacitance سے بھی کم آئے گی)

Example:
If two capacitors $C_1 = 4\mu F$ and $C_2 = 6\mu F$ are connected in series with a 12V battery as shown



Then

i. $C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{4\times6}{4+6} = \frac{24}{10} = 2.4\mu F$

ii. $Q_1 = Q_2 = Q = C_{eq}V = 2.4 \times 12\mu C = 28.8\mu C$ and $\frac{Q_1}{Q_2} = 1$

iii. $V_1 = \frac{C_2}{C_1+C_2}V = \frac{6}{4+6} \times 12 = 7.2V$

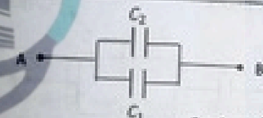
iv. $V_2 = \frac{C_1}{C_1+C_2}V = \frac{4}{4+6} \times 12 = 4.8V$ and $\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{3}{2}$

v. $V_1 + V_2 = 7.2 + 4.8 = 12V = V$

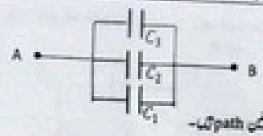
PARALLEL COMBINATION OF CAPACITORS

If capacitors are connected are connected side by side such that same potential difference is applied across all of them then this combination is known as parallel combination.

(اگر ایک پوائنٹ سے دوسرے پوائنٹ تک charges کو جانے کے لیے ایک سے زیادہ path موجود ہوں تو capacitors آپس میں parallel میں ہوں گے)



A سے B تک دو path ہیں۔



A سے B تک تین path ہیں۔

➢ When capacitors are connected in parallel then voltage across each capacitor is same

$$V_1 = V_2 = V \quad \text{And} \quad \frac{V_1}{V_2} = 1$$

When capacitors are connected in parallel total charge is divided among the capacitors

$$Q = Q_1 + Q_2$$

As V is same so

$Q \propto C$ and $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$ (جس کی capacitance زیادہ ہو گی اس پر زیادہ چارج store ہو گا)

$P.E \propto C$ and $\frac{P.E_1}{P.E_2} = \frac{C_1}{C_2}$ (جس کی capacitance زیادہ ہو گی اس میں زیادہ انرجی store ہو گی)

Charge divider Rule

If two capacitors are connected in parallel then total charge Q is divided among them as

$$Q_1 = \frac{C_1}{C_1 + C_2}Q \quad \text{and} \quad Q_2 = \frac{C_2}{C_1 + C_2}Q$$

As V is same and P.E $= \frac{1}{2}CV^2$ so (جس کی capacitance زیادہ ہوگی اس میں زیادہ انرجی store ہوگی)

Equivalent Capacitance:

If 'n' no. of capacitors are connected in parallel then

$$C_{eq} = nC$$

(سب سے پہلے دیکھیں اگر capacitors کی ویلیو same ہے تو capacitance کو تعداد سے multiply کر دیں)

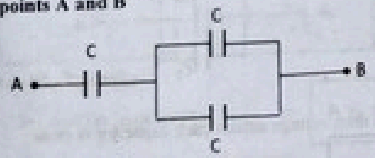
$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_n$$

(اگر different values کے capacitors ہوں تو سب capacitances کا sum کر دیں)

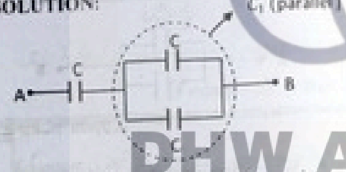
- $C_{eq} > C_{max}$
- To increase the capacitance capacitors are connected in parallel.

PRACTICE EXAPLES

EXAMPLE1:
Find equivalent capacitance between the points A and B



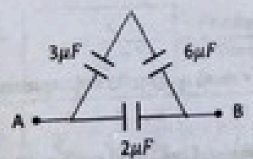
SOLUTION:



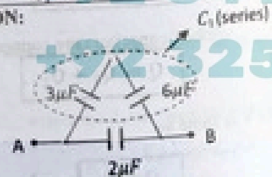
$C_1 = C + C = 2C$

$C_{eq} = \frac{C_1 \times C}{C_1 + C} = \frac{2C \times C}{2C + C} = \frac{2C}{3}$

EXAMPLE2:
Find equivalent capacitance between the points A and B



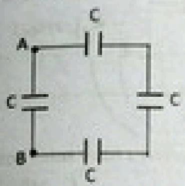
SOLUTION:



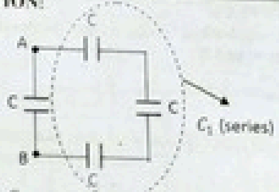
$C_1 = \frac{3 \times 6}{3 + 6} = 2$

$C_{eq} = C + C_1(parallel) = 2 + 2 = 4\mu F$

EXAMPLE3:
Find equivalent capacitance between the points A and B



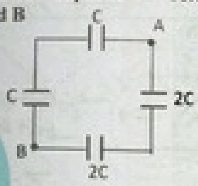
SOLUTION:



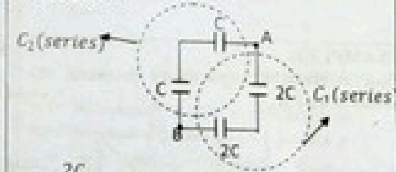
$C_1 = \frac{C}{3}$ (capacitors have same value)

$C_{eq} = C + C_1(parallel) = C + \frac{C}{3} = \frac{4C}{3}$

EXAMPLE4:
Find equivalent capacitance between the points A and B



SOLUTION:

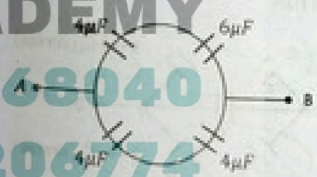


$C_1 = \frac{2C}{2} = C$ (capacitors have same value)

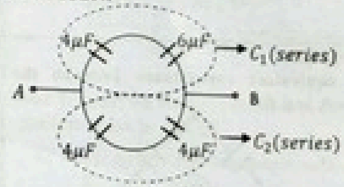
$C_1 = \frac{C}{2}$ (capacitors have same value)

$C_{eq} = C + \frac{C}{2}(parallel) = \frac{3C}{2}$

EXAMPLE5:
Find equivalent capacitance between the points A and B



SOLUTION:

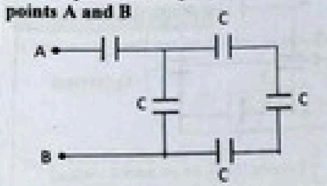


$C_1 = \frac{4 \times 6}{4 + 6} = 2.4$

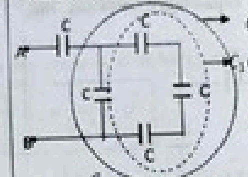
$C_2 = \frac{4\mu F}{2}$ (capacitors have same value)

$C_{eq} = C_1 + C_2 = 2 + 2.4 = 4.4\,\mu F$

EXAMPLE6:
Find equivalent capacitance between the points A and B



SOLUTION:

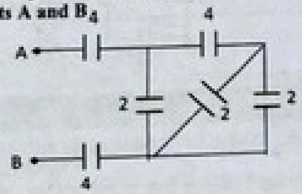


$C_1 = \frac{C}{3}$ (capacitors have same value)

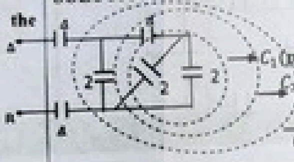
$C_2 = C + C_1 = C + \frac{C}{3} = \frac{4C}{3}$

$C_{eq} = \frac{C \times \frac{4C}{3}}{C + \frac{4C}{3}} = \frac{4C}{7}$

EXAMPLE7:
Find equivalent capacitance between the points A and B



SOLUTION:



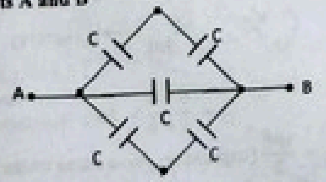
$C_1 = 2 + 2 = 4$

$C_2 = \frac{4}{2} = 2$ (same values)

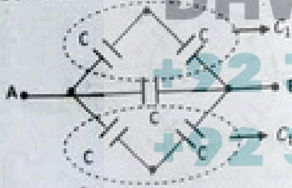
$C_3 = 2 + 2 = 4$

$C_{eq} = \frac{4}{3}$

EXAMPLE6:
Find equivalent capacitance between the points A and B



SOLUTION

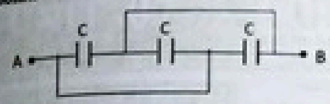


$C_1 = \frac{C}{2}$ (capacitors have same value)

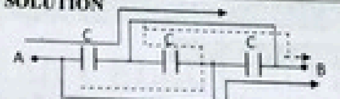
$C_2 = \frac{C}{2}$ (capacitors have same value)

$C_{eq} = C + \frac{C}{2} + \frac{C}{2} = 2C$

EXAMPLE6:
Find equivalent capacitance between the points A and B



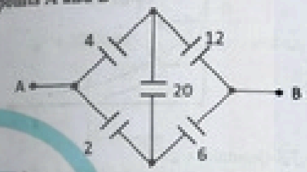
SOLUTION



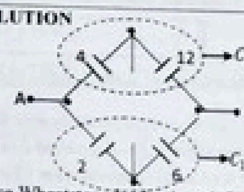
There are three path from A to B hence three capacitors are in parallel

$C_{eq} = \frac{C}{3}$ (capacitors have same value)

EXAMPLE6:
Find equivalent capacitance between the points A and B



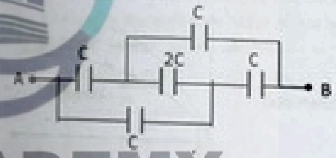
SOLUTION

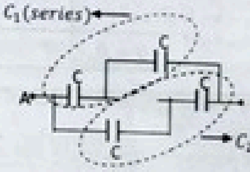


Since Wheatstone bridge is satisfied so 20F capacitance can be deleted

$C_1 = \frac{4 \times 12}{4+12} = 3$ and $C_2 = \frac{2 \times 6}{2+6} = 1.5$

$C_{eq} = 3 + 1.5 = 4.5$





Since Wheatstone bridge is satisfied so 2C capacitance can be deleted

$C_1 = \frac{C}{2}$ (capacitors have same value)

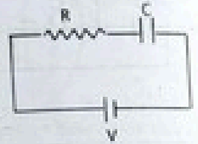
$C_2 = \frac{C}{2}$ (capacitors have same value)

$C_{eq} = \frac{C}{2} + \frac{C}{2} = C$

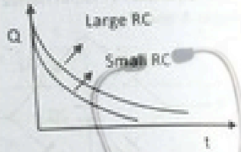
CHARGING AND DISCHARGING OF CAPACITORS

Charging of capacitor :

- To charge the capacitor , capacitor is connected with a voltage source(battery) as shown in the figure below
- Charging of a capacitor continues until the potential difference of capacitor becomes equal to the potential difference of source.



- At any instant the charge on the plates of capacitor is given as $Q = Q_o(1 - e^{-t/RC})$
- With the passage of time charging of capacitor slowly.
- Charge increases exponentially with time



- **Time constant:** ***Time required by a capacitor to deposit 0.63 or 63% equilibrium charge.***

$t = RC$ اگر کوئی دو Quantities دی گئی ہوں تو ان کی relation سے تیسری معلوم کریں

Unit: It's unit is second.

Discharging of Capacitor:

- To discharge the capacitor at any instant 't' charge on the capacitor is given as

$$Q = Q_o e^{-t/RC}$$

- Initially discharging is fast, later on it slow down and then stop
- Charge decreases exponentially with time.
- **Time constant**:

Time required to discharge 0.63 or 0.63% of equilibrium charge is called time constant.

$t = RC$ اگر کوئی دو Quantities دی گئی ہوں تو ان کی relation سے تیسری معلوم کریں

- Unit of RC is second $ohm \times forced = sec$
- After time equal to time constant ($t = RC$), 63% capacitor is discharged and remaining charge on the capacitor is 37%.

Interesting Application
The charging and discharging of a capacitor enables some windshield wipers of cars to be used intermittently during a light drizzle in this mode of operation the wipers remain off for a while and turn on briefly. The timing of on-off cycle is determined by time constant of visitor-capacitor combination.

UNIT 07 ⟩⟩ CURRENT ELECTRICITY

ELECTRIC CURRENT:

- ***Rate of flow of charge is called electric current or charge passing through cross section area of the conductor per unit time is called current.***
- Average current is given as

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

- Voltage ہر coulomb پر چارج کی انرجی کو ظاہر کرتا ہے۔
- Current ہر سیکنڈ میں گزرنے والے چارج کی مقدار کو ظاہر کرتا ہے۔

- Slope of Q-t graph represent the current

- Current is a scalar quantity.
- Current is a base quantity and its SI unit is ampere ($A = Cs^{-1}$)
- Current is one ampere if one coulomb charge is passing through conductor in one second.
- If 'n' is number of electrons (or protons) passing through a point in time t then $Q = ne$ and average current is

$$I = \frac{ne}{t}$$

یاد رکھیں
Current کی direction ہونے کے باوجود یہ ایک scalar quantity ہے۔

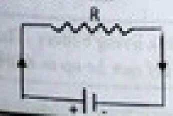
Note
One ampere current means 6.25×10^{18} electrons are passing through a conductor in one second

Charge Carriers:

Substance	Metals	Electrolytes	Gases	Semi-conductors
Charge carriers	Free electron	Positive and negative electron	Ions and free e^-	Free electrons and holes

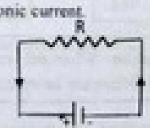
Conventional Current:

Equivalent current due to flow of positive charge carriers from high potential to low potential is called conventional current.



Electronic current:

Current due to flow of electrons (negative charge carriers) from low potential to high potential is called electronic current.

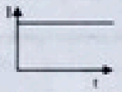
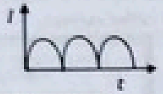
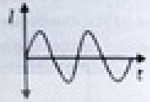


یاد رکھیں

جب بھی ہم electronic current کی بجائے صرف current کی بات کرتے ہیں تو اس سے مراد Conventional Current ہوتا ہے۔

Other Types of Current:

(i) Steady current (D.C) (ii) Pulsating (D.C) (iii) Alternating Current (A.C)

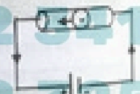
Current though metallic conductor:

- In metallic conductors charge carriers are free electrons.
- Free electrons are in state of random motion like gas molecules, electrons collide with each other and with lattice atoms and thus they change their direction.
- Thermal velocity of electrons is several 100 km/s
- **When no potential difference is applied across conductor**
  - Average velocity of electrons is zero.
  - Net flow of electrons is zero.
  - Current through conductor is zero.

- **When p.d.is applied across the conductor.**
  - An electric field is produced in the conductor which exerts force on electrons in opposite direction of electric field.
  - Electrons are still in state of random motion but flow of electrons towards high potential (+ve) is greater than flow of electrons towards low potential (−ve) and thus net flow is not zero.
  - Electrons are drifted towards high potential and an electric current passes through conductor.

Drift velocity:

Average velocity gained by electrons when a potential difference is applied across the conductor is called drift velocity.



- Drift velocity is of the order of $10^{-3} m/s$ or $1 mms^{-1}$
- Drift velocity of electrons is always opposite to direction of electric field.

$V_d = \frac{I}{neA}$ OR $V_d = \frac{V}{RneA}$ (n is number of charge carriers per unit volume)

Information: When eel senses danger ,it turns itself into a living battery .The potential difference between head and tail of ***an electric eel can be up to 600V.***

Source of current:

A device which maintains a constant Potential difference across the two ends of a conductor is called source of current.
Source of current converts some non-electrical energy into electrical energy.

Ideal current source:
A current source which maintains a constant current irrespective of load resistance is called ideal current source.
Its internal resistance is infinite

Source of current	Cell / Battery	Generator	Thermocouple	Solar cell
Converts	Chemical energy into electrical energy	Mechanical energy into electrical energy	Heat energy into electrical energy	Light energy into electrical energy

Ideal voltage source:
A voltage source whose output voltage is independent of current drawn from it is called ideal voltage source.
Its internal resistance is zero.

یاد رکھیں

کسی بھی conductor میں current گزرنے کے لیے دو شرائط ہیں:
i. Conductor میں charge carriers موجود ہوں۔
ii. Conductor کے across پوٹینشل difference موجود ہو۔

EFFECTS OF CURRENT

Heating effect:

- ***Current passing through a conductor produces heat in the conductor.***
- When current passes through conductor electrons collide with atoms and transfer some energy to atoms thus average K.E of atoms increases and temperature of conductor increases.
- *Applications:* Electric heater, electric stove, electric kettle, electric iron, filament bulb, toaster etc.

Joule's law of heating:

When current I is passing through conductor of resistance R for time t then heat produced in conductor is given as

$H = I^2Rt$

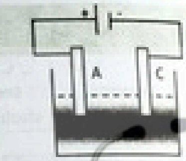
دلچسپ معلومات

جن devices میں current پاس کرتے ہیں ان کے ساتھ کم Resistance والی تاریں لگائی جاتی ہیں تاکہ ان میں کم heat پیدا ہو اور وہ burn نہ ہوں۔

Magnetic effect:

- ***Current produces through the conductor produces magnetic field around the conductor.***
- Strength of magnetic field depends upon amount of current.
- Pattern of magnetic field depends upon shape of conductor.
- ***Applications***: Voltmeter, ammeter, galvanometer, motors, electromagnets, speaker etc.

Chemical effect:



Current passing through electrolyte produces chemical changes in it.

Electrolytes: liquids which can conduct electricity are called electrolytes.

Electrodes: rods, plates or wires which load current into electrolyte and out of electrolyte are called electrodes.

Anode: Electrode connected to +ve terminal of battery.

Cathode: Electrode connected to –ve terminal of battery.

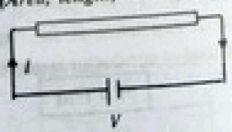
Voltmeter: vessel containing the electrolyte and electrodes is called voltmeter. • When $CuSO_4$ is dissolved in water it splits up into Cu^{+2} and SO_4^{-2} ions. $CuSO_4 \longrightarrow Cu^{+2} + SO_4^{-2}$ **At Cathode:** Cu^{+2} ions move towards the cathode and get deposit there $Cu^{+2} + 2e^- \longrightarrow Cu$	**At Anode:** SO_4^{-2} ions move towards the anode and remove one cell atom of anode. $SO_4^{-2} + Cu \longrightarrow CuSO_4 + 2e^-$ ***Electroplating:*** *Process of coating a thin layer of some expensive metal on an article of cheap metal is called electroplating.* یاد رکھیں: metal کو dissolve کرتا ہے anode اور deposit کرتا ہے cathode پر کرتا ہے۔

OHM'S LAW

"Current passing through conductor is directly proportional to potential difference applied across the conductor provided that physical conditions (Area, length, volume, temperature, strain etc) are constant"

$I \propto V \quad or \quad I = \frac{V}{R}$

$\Rightarrow \boxed{V = IR}$



Where R is constant and known as resistance of conductor.

Resistance:

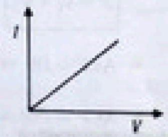
Resistance is measure of opposition in the flow of electrons due to their continuous bumping with atoms of the lattice.

$R = \frac{V}{I}$

> Resistance of a conductor is independent of applied voltage and current passing through the circuit.

- SI unit of resistance is ohm $\left(ohm = \frac{volt}{ampere}\right)$
- Resistance is one ohm if one ampere current passes through conductor in one second.
- In terms of base units the unit of resistance is $kgm^2s^{-3}A^{-2}$ $(dimensions = [ML^2T^{-3}A^{-2}])$

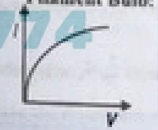
Ohmic Devices:

Devices which obey the Ohm's law are called ohmic devices. • Conductance is reciprocal of resistance $Conductance = \frac{1}{resistance}$ • Resistance and conductance of an ohmic device remains constant. • Resistors and metallic wires for constant temperature are ohmic.	I-V graph for Ohmic devices is a straight line.  Slope of I-V graph represents the conductance.

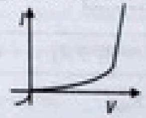
Non-Ohmic Devices:

- Devices which do not obey Ohm's law are called non-ohmic devices.
- I-V graph for non-ohmic devices is not a straight line (non-linear).
- Slope of I-V graph represent the conductance.
- Resistance and conductance of a non-ohmic device does not remains constant.
- Filament bulb, diodes, discharge tubes, transistors, capacitors, inductor etc. are non-ohmic devices.

Filament Bulb:

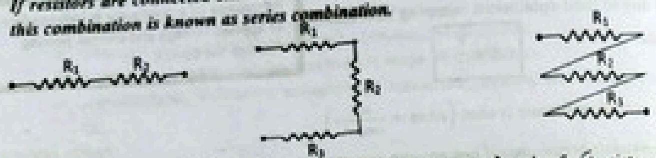


Semiconductor Diode:



1. Series combination:

If resistors are connected end to end such that same current is passing through all of them this combination is known as series combination.



(اگر resistors ایک ہی path میں connected ہوں تو resistors سیریز میں ہوں گے)

- Current passing through each resistor is same $\boxed{I_1 = I_2 = I}$ and $\boxed{\frac{I_1}{I_2} = 1}$

$$\boxed{I_1 = I_2 = \frac{V}{R_{eq}}}$$

(کسی بھی resistor سے گزرنے والا current معلوم کرنے کے لیے پہلے R_{eq} معلوم کریں)

- Voltage is divided among the resistors

$$\boxed{V = V_1 + V_2}$$

★ As $V = IR \Rightarrow \boxed{V \propto R}$ (جس کی resistance زیادہ ہوگی اس کے across voltage drop زیادہ ہوگا)

★ As $P = I^2R \Rightarrow \boxed{P \propto R}$ (جس کی resistance زیادہ ہوگی اس میں Power بھی زیادہ خرچ ہوگی)

Voltage Divider Rule:

If two resistance R_1 and R_2 are in series with voltage V then

$$\boxed{V_1 = \frac{R_1}{R_1 + R_2}V} \text{ and } \boxed{V_2 = \frac{R_2}{R_1 + R_2}V}$$

Example: Two resistances $R_1 = 2\Omega$ and $R_2 = 4\Omega$ are connected in series with a 12V battery then voltage drop across 4Ω resistor will be

(a) 2V (b) 4V (c) 6V (c) 8V

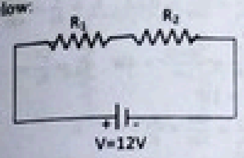
Solution: $V_2 = \frac{R_2}{R_1 + R_2}V = \frac{4}{4+2} \times 12 = 8V$

Equivalent Resistance:

If 'n' number of resistors are connected in series then

1. $\boxed{R_{eq} = nR}$ (اگر سب سے پہلے دیکھیں کہ resistance برابر ہیں تو resistance کو تعداد سے multiply کر دیں)
2. $\boxed{R_{eq} = R_1 + R_2 + \cdots + R_n}$ (اگر مختلف values کی resistance ہوں تو سب resistances کا sum کر دیں)
3. $R_{eq} > R_{max}$
4. To increase the resistance, resistors are connected in series.

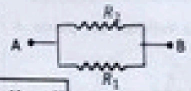
Example:
If two resistors $R_1 = 2\Omega$ and $R_2 = 4\Omega$ are connected in series are as shown in the figure below:

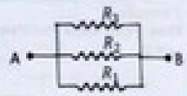


1. $Req = R_1 + R_2 = 2 + 4 = 6\Omega$
2. $I = I_1 = I_2 = \frac{V}{Req} = \frac{12}{6} = 2A$
3. $V_1 = \frac{R_1}{R_1+R_2}V = \frac{2}{2+4}(12) = \frac{2}{6}(12) = 4V$
4. $V_2 = \frac{R_2}{R_1+R_2}V = \frac{4}{2+4}(12) = \frac{4}{6}(12) = 8V$
5. $\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{2}{4} = \frac{1}{2}$
6. $\frac{I_1}{I_2} = \frac{2}{2} = 1$
7. $P_1 = I^2R_1 = (2)^2(2) = 4 \times 2 = 8W$
   $P_2 = I^2R_2 = (2)^2(4) = 4 \times 4 = 16W$

Parallel Combination:

If resistors are connected side by side such that same P.d is applied across all of them then this combination is known as **parallel** combination.





- Voltage across each resistor is same. $\boxed{V_1 = V_2 = V}$
- Current is divided $\boxed{I = I_1 + I_2}$

★ As $\boxed{I = \frac{V}{R}} \Rightarrow I \propto \frac{1}{R}$ (جس کی resistance زیادہ ہوگی اس میں سے کرنٹ کم گزرے گا)

★ As $\boxed{P = \frac{V^2}{R}} \Rightarrow P \propto \frac{1}{R}$ (جس کی resistance زیادہ ہوگی اس میں Power کم خرچ ہوگی)

Current Divider Rule:

If two resistors R_1 and R_2 are connected in parallel with total current I then current through each resistor is given as

$$\boxed{I_1 = \frac{R_2}{R_1 + R_2}I} \text{ and } \boxed{I_2 = \frac{R_1}{R_1 + R_2}I}$$

Equivalent Resistance:

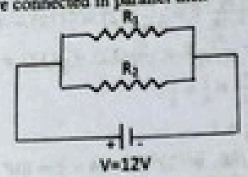
If 'n' number of resistors are connected in series then

1. $\boxed{R_{eq} = \frac{R}{n}}$ (اگر سب سے پہلے دیکھیں کہ resistors کی values برابر ہیں تو ان کو تعداد پر divide کر دیں)
2. $R_{eq} = \frac{R_1R_2}{R_1+R_2} = \frac{\text{product of resistances}}{\text{sum of resistanc e}}$ (اگر دو مختلف values کے resistors ہوں)
3. $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$ (اگر زیادہ different values کے capacitor ہوں تو اس فارمولے کو use کریں)
4. To decrease the resistance, resistors are connected in parallel.
5. $\boxed{R_{series} = n^2 R_{parallel}}$

R_{series} = Equivalent resistance when resistors are connected

$R_{parallel}$ = Equivalent resistance when resistors are connected in parallel.

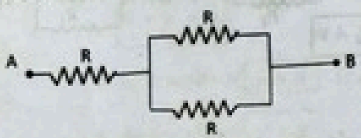
Example:
If two resistors $R_1 = 100\Omega$ and $R_2 = 400\Omega$ are connected in parallel then



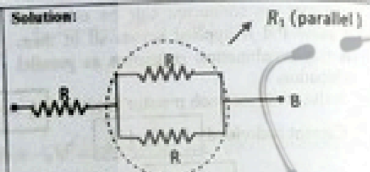
1. $Req = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{100 \times 400}{500} = 80\Omega$
2. $V_1 = V_2 = V = 20V$
3. $I_1 = \frac{V}{R_1} = \frac{20}{100} = 0.2A$, $I_2 = \frac{V}{R_2} = \frac{20}{400} = 0.05A$
4. $P_1 = \frac{V^2}{R_1} = \frac{20 \times 20}{100} = 4W$, $P_2 = \frac{V^2}{R_2} = \frac{20 \times 20}{400} = 1W$
5. $\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{400}{100} = \frac{4}{1}$
6. $\frac{V_1}{V_2} = \frac{20}{20} = 1$

PRACTICE EXAMPLES

Example 1:
Find equivalent resistance between the points A and B



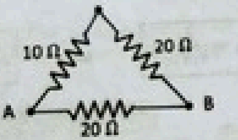
Solution:



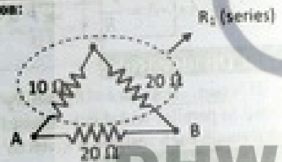
$R_1 = \frac{R}{2}$ (resistors have same values)

$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$

Example 2:
Find equivalent resistance between the points A and B



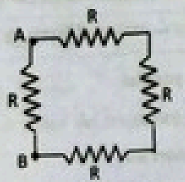
Solution:



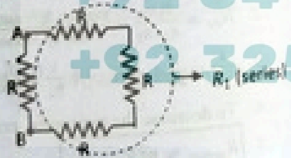
$R_1 = 10 + 20 = 30\Omega$

$R_{eq} = \frac{20 \times 30}{20 + 30} = 12\Omega$

Example 3:
Find equivalent resistance between the points A and B



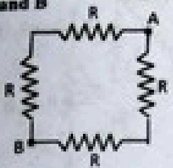
Solution:



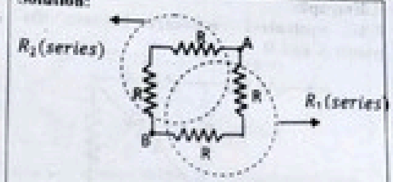
$R_1 = R + R + R = 3R$

$R_{eq} = \frac{R \times 3R}{R + 3R} = \frac{3R}{4}$

Example 4:
Find equivalent resistance between the points A and B



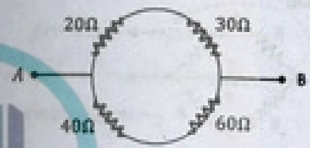
Solution:



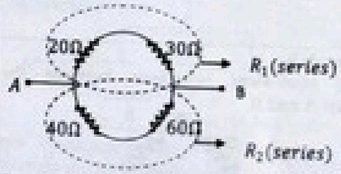
$R_1 = R + R = 2R$ and $R_1 = R + R = 2R$

$R_{eq} = \frac{2R}{2} = R$

Example 5:
Find equivalent resistance between the points A and B



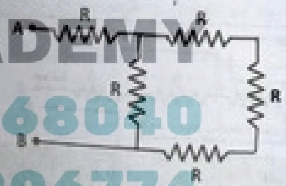
Solution:



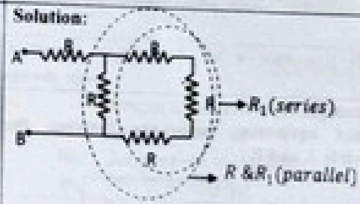
$R_1 = 20 + 30 = 50\Omega$ and $R_2 = 40 + 60 = 100\Omega$

$R_{eq} = \frac{50 \times 100}{50 + 100} = \frac{100}{3}$

Example 6:
Find equivalent resistance between the points A and B



Solution:

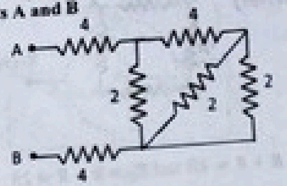


$R_1 = R + R + R = 3R$

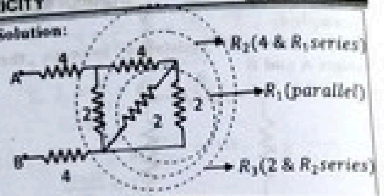
$R_2 = \frac{R \times 3R}{R + 3R} = \frac{3R}{4}$

$R_{eq} = R + \frac{3R}{4} = \frac{7R}{4}$

Example 7:
Find equivalent resistance between the points A and B



Solution:

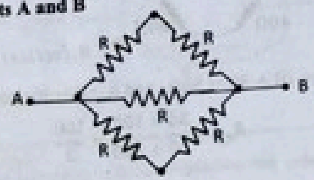


$$R_1 = \frac{2}{2} = 1 \text{ and } R_2 = 1 + 4 = 5$$

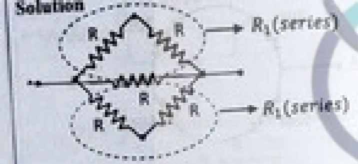
$$R_3 = \frac{2 \times 5}{2+5} = \frac{10}{7}$$

$$R_{eq} = 4 + 4 + \frac{10}{7} = \frac{66}{7}$$

Example 8:
Find equivalent resistance between the points A and B



Solution

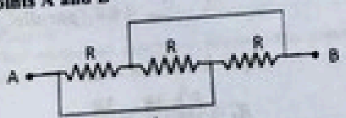


$$R_1 = R + R = 2R \text{ and } R_2 = R + R = 2R$$

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R}$$

$$R_{eq} = \frac{R}{2}$$

Example 9:
Find equivalent resistance between the points A and B

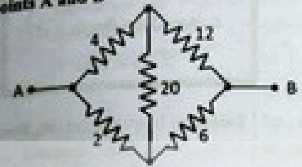


Solution

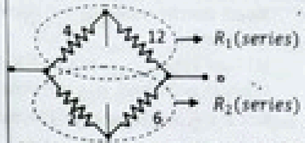
There are three path from A to B hence three resistors are in parallel

$$R_{eq} = \frac{R}{3} \text{(resisters have same value)}$$

Example 10:
Find equivalent resistance between the points A and B



Solution

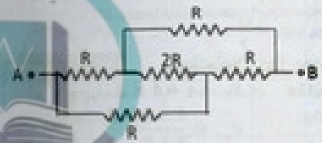


Since Wheatstone bridge is satisfied so 20 ohm resister can be deleted

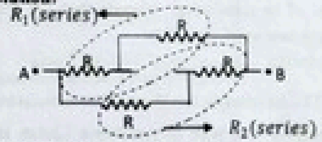
$$R_1 = 4 + 12 = 16 \text{ and } R_2 = 2 + 6 = 8$$

$$R_{eq} = \frac{8 \times 16}{8 + 16} = \frac{16}{3}$$

Example 11:
Find equivalent resistance between the points A and B



Solution:



Since Wheatstone bridge is satisfied so 2R resistance can be deleted

$$R_1 = R + R = 2R \text{ and } R_2 = R + R = 2R$$

$$R_{eq} = \frac{2R}{2} = R$$

TEMPERATURE DEPENDENCE OF RESISTIVITY

Resistance:

It is found that resistance of a conductor is directly proportional to length of conductor and inversely proportional to cross-sectional area of the conductor $R \propto \frac{L}{A}$ or $R = \frac{\rho L}{A}$	**Resistance is property of a wire and it depends upon.** i. Length of conductor ii. Area of conductor iii. Temperature of conductor iv. Nature of material

Relation with length of wire	Relation with area of wire	Relation with radius of wire	Relation with diameter of wire
$R \propto L$	$R \propto \frac{L}{A}$	$R \propto \frac{1}{r^2}$	$R \propto \frac{1}{d^2}$

Example: If a wire is stretched to twice of its length then its resistance will become
(a) Double (b) Half
(c) Four Times (d) One Fourth

Solution: If length is doubled, area will become half $R \propto \frac{L}{A}$ so resistance will become 4-times

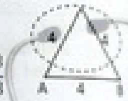
یاد رکھیں: جب wire کو stretch کریں تو length جتنے گنا بڑھے Area اتنے گنا کم ہو جائے۔ R اس کا time بڑھ جائے گا

Example: If a wire of resistance R is cut into three equal parts and these parts are connected in parallel then its equivalent resistance will become
(a) R (b) $\frac{R}{3}$ (c) $\frac{R}{9}$ (d) 3R

Solution: If wire is cut into three equal parts then resistance of each part is $\frac{R}{3}$ by connecting in parallel $R_{eq} = \frac{R/3}{3} = \frac{R}{9}$

Example: If a wire of resistance 12 ohm is bent along the sides of an equilateral triangle then the resistance between its any two vertices will be
(a) 6 ohm (b) $\frac{4}{3}$ ohm (c) $\frac{8}{3}$ ohm (d) 4 ohm

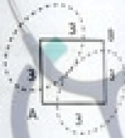
Solution:
$R_1 = 4 + 4 = 8$
$R_{eq} = \frac{4 \times 8}{4+8} = \frac{8}{3}$



NOTE: As triangle has three sides so resistance of each side $= \frac{12}{3} = 4$

Example: If a wire of resistance 12ohm is bent along the sides of a square then the resistance along diagonal will be
(a) 6 ohm (b) $\frac{4}{3}$ ohm (c) $\frac{8}{3}$ ohm (d) 4 ohm

Solution:
$R_1 = 4 + 4 = 8$
$R_{eq} = \frac{4 \times 8}{4+8} = \frac{8}{3}$



NOTE: As square four sides so resistance of each side $= \frac{12}{4} = 3$

Example: If a wire of resistance 20 ohm is bent along the circle then the resistance along diameter will be
(a) 6 ohm (b) $\frac{4}{3}$ ohm (c) $\frac{8}{3}$ ohm (d) 4 ohm

Solution:
$R_{eq} = \frac{10}{2} = 5$

Conductance:

➢ ***Reciprocal of resistance is called conductance*** $G = \frac{1}{R} = \frac{A}{\rho L}$ ➢ Unit of conductance is ohm^{-1} or mho or simen	***Conductance is property of a wire and it depends upon.*** i. Length of conductor ii. Area of conductor iii. Temperature of conductor iv. Nature of material

Relation with length of wire	Relation with area of wire	Relation with radius of wire	Relation with diameter of wire
$G \propto \frac{1}{L}$	$G \propto A$	$G \propto r^2$	$G \propto d^2$

Resistivity or Specific Resistance:

➢ ***Resistance of a meter cube of a material is called resistivity or specific resistance.*** $\rho = \frac{RA}{L}$ ➢ SI unit of resistivity is ohm-m ($kgm^3s^{-3}A^{-2}$) and dimensions are $[ML^3T^{-3}A^{-2}]$	**Dependence:** ➢ Resistivity is property of material and it is independent of length, area or dimensions of conductor. ➢ Resistivity only depends upon temperature and nature of material.

یاد رکھیں
Resistivity ایک proportionality کانسٹنٹ ہے اور کسی بھی فارمولے میں
proportionality کانسٹنٹ اس فارمولے کی باقی quantities پہ depend نہیں کرتا۔

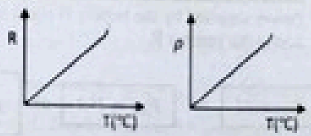
Conductivity:

➢ ***Reciprocal of resistivity is called conductivity.*** $\sigma = \frac{1}{\rho} = \frac{L}{RA}$ ➢ Its SI unit $ohm^{-1}m^{-1}(kg^{-1}m^{-3}s^3A^2)$	**Dependence:** ➢ Conductivity is property of material and it is independent of length, area or dimensions of conductor. ➢ Conductivity only depends upon temperature and nature of material.

Temperature dependence:

By increasing temperature, average K.E of atoms increases due to which the amplitude of vibrations of atoms increases thus probability of collisions of electrons with atoms increases. Since resistance is due to collision of electrons with atoms hence resistivity or resistance of conductor increases by increasing temperature.

➢ Resistance or resistivity of metals increase linearly with temperature.



Temperature Coefficient of Resistance:

Fractional change in resistance per Kelvin is called temperature coefficient of resistance.

$$\alpha = \frac{R_t - R_o}{R_o t}$$

- Its SI unit is K^{-1}.
- It only depends upon nature of material.
- For all metals $(Cu, Al, Fe\ etc.)$ α is positive which means by increasing temperature their resistance increases (conductance decreases).
- For semi-conductors insulators and electrolytes (C, Si, Ge) α is negative which means by increasing temperature their resistance decreases (and conductance increases)

Temperature Coefficient of Resistivity:

Fractional change in resistivity per Kelvin is called temperature coefficient of resistivity.

$$\alpha = \frac{\rho_t - \rho_o}{\rho_o t}$$

- Its SI unit is K^{-1}.

یاد رکھیں
α کی value زیادہ ہونے کا مطلب ہے کہ اگر temperature میں تھوڑا سا change آئے تو Resistance میں بہت زیادہ Change آئے گا۔

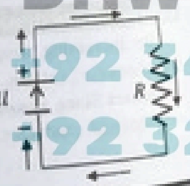
Electrical Power

Electrical power:

"Energy supplied by cell or battery per unit time is called electrical power of the battery or cell."

$$P = \frac{Energy\ supplied}{Time} \quad \text{or} \quad P = VI$$

Charge Δq move from high potential to low potential and dissipates its energy across resistance R and comes to low potential. Battery supplies the energy to charge and move it from low potential to high potential.



Power dissipated across resistor:

Power supplied by the battery is equal to power dissipated across the resistor R

$$P_{dis} = VI \quad \text{or} \quad P_{dis} = I^2R \quad \text{or} \quad P_{dis} = \frac{V^2}{R}$$

اگر resistor سیریز میں لگے ہوں تو $P = I^2R$ اور اگر resistor پیرالل میں لگے ہوں تو $P = \frac{V^2}{R}$ کو prefer کریں۔

Series Combination of Power:

If 'n' number of devices are connected in series then,

1. $P_{eq} = \frac{P}{n}$ (سب سے پہلے دیکھیں اگر device کی ویلیو same ہے تو Power کو تعداد پر divide کر دیں)
2. $P_{eq} = \frac{P_1P_2}{P_1 + P_2} = \frac{product\ of\ power}{sum\ of\ power}$ (اگر دو different value کے devices ہوں تو P_{eq} معلوم کرنے کے لیے use کریں)
3. $\frac{1}{P_{eq}} = \frac{1}{P_1} + \frac{1}{P_2} + \cdots + \frac{1}{P_n}$ (اگر زیادہ value کے devices ہوں تو P_{eq} کے لیے use کریں)
4. $P_{eq} < P_{min}$ (series میں اگر کوئی ایک device بھی off ہو جائے تو باقی سب بھی off ہو جائیں گے)
5. To decreases the power devices are connected in series.

Example: Two filament bulbs having power rating 100 W are connected in series as shown in the figure below. Then equivalent power will be?

(a) 70 W (b) 150 W (c) 240W (d) 300W

Solution: $P_{eq} = \frac{P_1P_2}{P_1+P_2} = \frac{100\times200}{100+200} = \frac{100 \times 200}{300} = \frac{200}{3} \sim 66\ W$

Parallel Combination:

If 'n' number of devices are connected in parallel then

$P_{eq} = nP$ سب سے پہلے دیکھیں اگر device کی ویلیو same ہے تو Power کو تعداد سے multiply کر دیں۔

1. $P_{eq} = P_1 + P_2 + \cdots P_n$ اگر different value کے devices ہوں تو سب کی power کا sum کر دیں۔

$P_{eq} > P_{max}$

2. To increase the power devices are connected in parallel.

Example:
Two filament bulbs having power rating 200 W and 500 W are connected in parallel Then equivalent power will be about?

(a) 140 W (b) 250 W (c) 350 W (d) 700 W

Solution:
$P_{eq} = P_1 + P_2$
$= 200 + 500 = 700\ W$

یاد رکھیں

1. ہمارے گھروں میں تمام devices پیرالل میں لگے ہوتے ہیں تاکہ سب کو برابر voltage مل سکے۔
2. تمام devices کی power rating ان کی ضرورت کے مطابق ہوتی ہے۔
3. جس device کی power زیادہ ہوتی ہے اس کی resistance کم ہوتی ہے۔ $P = \frac{V^2}{R}$ or $P \propto \frac{1}{R}$

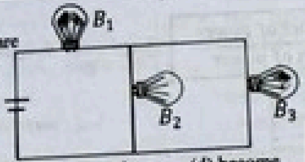
مثال کے طور پر زیادہ power والے filament bulb کے filament کی resistance کم ہوتی ہے یعنی thick filament ہوتا ہے۔

$$P \propto thickness\ of\ filament$$

4. جس device کی power زیادہ ہوگی وہ زیادہ current draw کرے گا۔ $P = VI$ or $P \propto I$

Parallel میں کوئی device کو add کرنے یا remove کرنے سے / on / off کرنے سے یا burn ہونے سے (open circuit) ہو جانے سے parallel میں لگے باقی devices پر کوئی فرق نہیں پڑتا۔

Example:
If three bulb B_1, B_2 and B_3 are connected with a battery as shown in the figure. If B_1 is burnt then what is effect on brightness of B_2 and B_3.



(a) increases (b) decreases (c) remain same (d) become zero

Solution:
If B_1 is burnt then it will break the circuit and stop the flow of current thus brightness of B_2 and B_3 becomes zero.

B_1 کے burn ہونے سے circuit ٹوٹ جائے گا اور current کا flow رک جائے گا۔ جس کی وجہ سے B_2 اور B_3 بھی بند ہو جائیں گے۔

اگر B_2 کے burn ہو جانے سے B_3 کی brightness پر کوئی فرق نہیں پڑے گا لیکن circuit کی equivalent resistance بڑھ جانے کی وجہ سے total current کم ہو جائے گا اور B_1 کی brightness کم ہو جائے گی۔

کوئی بھی device add یا remove کرنے سے series میں لگے device پر بہت فرق پڑتا ہے۔
اگر R بڑھ جائے تو total current کم ہو جائے گا اور brightness کم ہو جائے گی۔
اگر R کم ہو جائے تو total current بڑھ جائے گا اور brightness بڑھ جائے گی۔

ELECTROMOTIVE FORCE

Energy supplied by battery per unit charge is called electromotive force or EMF of battery.

$$EMF = \frac{Energy\ supplied}{Charge}$$

یاد رکھیں Battery ہر Coulomb چارج کو جتنی Energy مہیا کرتی ہے اسے EMF کہا جاتا ہے۔ مثال کے طور پر 12V کی Battery ہر Coulomb چارج کو 12J انرجی مہیا کرے گی۔

- It is a scalar quantity.
- Its SI unit is volt($Jc^{-1} = kgm^2s^{-3}A^{-1}$)
- Its dimensions are $[ML^2T^{-3}A^{-1}]$

Internal Resistance:

Resistance due to presence of electrolyte between the electrodes is called internal resistance. It is denoted by 'r'.

- EMF and internal resistance act in series.

- Internal resistance of an ideal voltage source is zero.

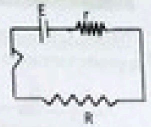
Dependence:

i. Distance between electrodes ($r \propto d$).
ii. Area between electrodes ($r \propto \frac{1}{A}$).
iii. Nature or concentration of electrolyte.
iv. Temperature

Three Important Cases:

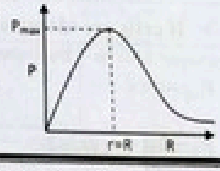
1. Closed Circuit:

Consider an external resistance R is connected across a battery of EMF 'E' and internal resistance 'r'.



- Current drawn from the battery is $I = \frac{E}{r+R}$
- Potential drop across external resistance or terminal Potential difference is $V_t = IR$
- Potential drop across internal resistance is 'I r'.
- Equation of cell/battery when it is discharging is $E = V_t + Ir$ and ($V_t < E$)
- Power dissipated in external resistance is $P = V_tI = I^2R = \frac{V_t^2}{R} = \frac{E^2R}{(r+R)^2}$
- Output power drawn from battery is maximum when R=r (internal resistance = external resistance)

$$P_{max} = \frac{E^2}{4r} \qquad P_{max} = \frac{E^2}{4R}$$



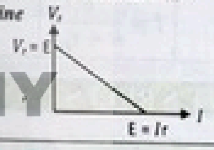
- Current drawn from battery is maximum when R= r

$$I_{max} = \frac{E}{2r} \qquad I_{max} = \frac{E}{2R}$$

NOTE: when battery is being charged i.e current is given to battery then

$V_t = E + Ir$ and $V_t > E$

The graph between Terminal Potential and current is a straight line



For x-intercept V= 0

So $E = Ir$ or $r = \frac{E}{I}$

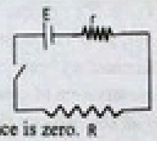
For y-intercept I= 0

$V_t = E$

گراف سے EMF اور Internal resistance دونوں معلوم کیے جا سکتے ہیں۔

2. Open Circuit:

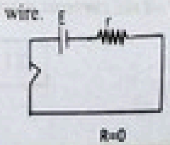
When no current is being drawn from battery or cell it is said to be open circuit.



- Current through circuit= 0
- Terminal potential difference is equal to ($V_t = E$).
- Potential drop across internal resistance and external resistance is zero.

3. Closed Circuit:

When two terminals of cell or battery are joined together by thick wire.



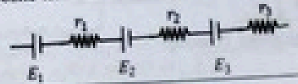
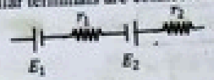
- Maximum current is drawn from battery

$$I_{max} = \frac{E}{r}$$

- Terminal potential difference is zero ($V = 0$).

GROUPING OF CELLS

1. Series Grouping:

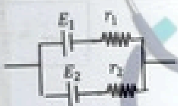
If opposite terminals are connected with each other. 	Equivalent EMF is given as $E_{eq} = E_1 + E_2 + E_3$ Equivalent internal resistance is given as $r_{eq} = r_1 + r_2 + r_3$
If similar terminals are connected with each other. 	Equivalent EMF is given as $E_{eq} = E_1 - E_2$ Equivalent internal resistance is given as $r_{eq} = r_1 + r_2$

➢ **If cells are identical**

Equivalent EMF is	Equivalent internal resistance	Current	Power
$E_{eq} = nE$	$r_{eq} = nr$	$I = \frac{E_{eq}}{R + r_{eq}}$	$P_{max} = n(\frac{E^2}{4r})$

2. Parallel Grouping:

If similar cathodes terminals are connected together at one point and anodes are connected together at other point.



Equivalent EMF is	Equivalent internal resistance	Total Current	Current through each cell
$E_{eq} = E$	$r_{eq} = \frac{r}{n}$	$I = \frac{E}{R + r_{eq}}$	$I_n = \frac{I}{n}$

KIRCHOFF'S RULES

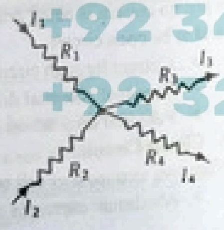
1. Kirchoff's First Rule:

➢ Kirchoff's first rule is also known as Kirchoff's current rule, point rate or junction rule.

➢ Kirchoff's 1st rule is manifestation of law of conservation of charge.

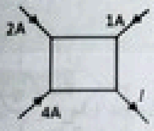
"Sum of all the currents flowing towards a point is equal to sum of all the currents flowing away from point."

"sum of all the currents meeting at a point is equal to zero"



$$\Sigma I = 0$$

Example: Figure shows a network of current. Then the current I will be



(a) 1A (b) 3A (c) 5A (d) 7A

Solution:

Total incoming current = 2 + 4 + 1 = 7A

Total out going current = 7A OR

" Sum of all the currents meeting at point is equal to zero" $\Sigma I = 0$

$2A + 4A + 1A - I = 0 \Rightarrow I = 7A$

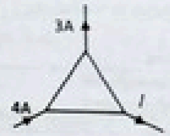
sign convention	Incoming current	Outing current
	+ve	−ve

Example:

Figure shows a network of currents then current I will be

(A) 1A (B) 4A

(C) 5A (D) 7A



Solution:

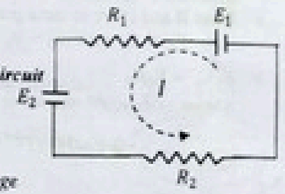
$4 - 3 - I = 0$

$\Rightarrow I = 1A$

2. 2nd Rule:

"Algebraic sum of all potential changes in closed circuit is equal to zero"

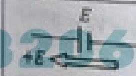
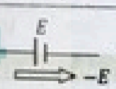
$$\Sigma V = 0$$



➢ Kirchoff's 2nd Rule is also known as *Kirchoff's Volage Rule (KVL)* and *Kirchoff's Loop Rule.*

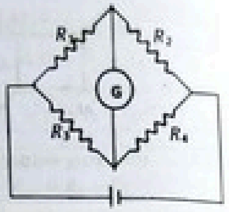
$+E_1 - IR_1 - E_2 - IR_2 = 0$

➢ Kirchoff's 2nd Rule is manifestation of law of conservation of energy.

FOR BATTERY		FOR RESISTORS	
Traversing from −ve to +ve	*Traversing from +ve to −ve*	*Traversing in direction of current*	*Traversing in opposite direction of current*
			

Wheatstone:

Wheatstone bridge is a circuit which is used to determine unknown resistance of a wire.



It's circuit diagram is shown in the figure:

Balancing Condition

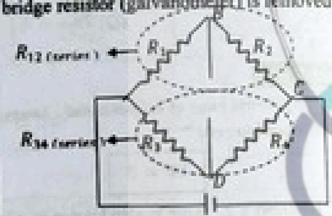
1. $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ or $\frac{R_1}{R_3} = \frac{R_2}{R_4}$

   Adjacent resistances کی ratio برابر ہونی چاہیے۔

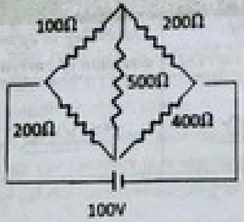
2. $R_1R_4 = R_2R_3$

   Opposite resistances کا Product برابر ہونا چاہیے۔

Under Balancing Condition

➤ No current passes through the galvanometer and shows zero deflection. ➤ Point B and D are at same potential. $V_B = V_D$ and $V_B - V_D = 0$ ➤ $R_1R_4 = R_2R_3$ relation اگر کوئی تین resistances معلوم ہوں تو اس سے چوتھی کو معلوم کر سکتے ہیں۔	To find equivalent resistance of the circuit bridge resistor (galvanometer) is removed  R_{12} and R_{34} are in parallel

Example:
Five resistors are connected with a 100V battery as shown in the figure below:



Solution: $R_1R_4 = R_2R_3$

$\Rightarrow 100 \times 400 = 200 \times 200$

$400 = 400$

Balancing condition is satisfied

➤ No current passes through 500Ω resistance.

$V_B = V_D$ or $V_B - V_D = 0$

Equivalent resistance:

$R_{12} = 100 + 200 = 300$ and $R_{34} = 200 + 400 = 600$

$$Req = \frac{300 \times 600}{300 + 600} = 200\Omega$$

UNIT 08 ›› ELECTROMAGNETISM

Electromagnetism:

➤ ***Branch of physics which deals with study of magnetic effects produced by motion of charges is called electromagnetism.***

➤ If a magnetic compass is placed near current carrying conductor, the magnetic field will deflect its direction.

➤ In 1820 Orested discovered that current passing through a conductor produces magnetic field around the conductor

Note: When a steady current is passing through a conductor then

Inside the conductor	Outside the conductor
$E \neq 0$ but $B = 0$	$B \neq 0$ but $E = 0$

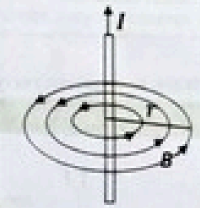
Magnetic Field Due To Current Carrying Straight Conductor

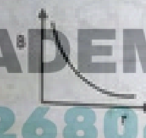
➤ Magnetic field produced by current carrying straight conductor is circular (concentric circles)

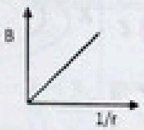
➤ Magnetic field lasts only as long as current is passing through conductor.

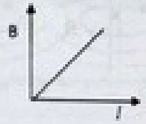
➤ If 'I' is current passing through conductor then at any distance 'r' from the conductor magnetic field given as

$$B = \frac{\mu_o I}{2\pi r} \Rightarrow B \propto I \text{ and } B \propto \frac{I}{r}$$









Where $\mu_o = 4\pi \times 10^{-7}\ Wbm^{-1}A^{-1}$ and known as permeability of free space.

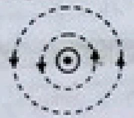
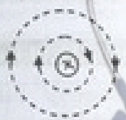
Note
➤ r is distance from the conductor and it is not radius of wire of conductor.

➤ Direction of magnetic field depends upon direction of current and it is determined by right hand rule.

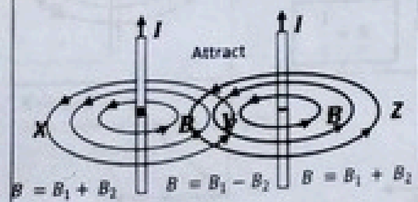
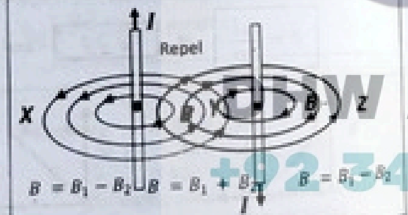
"Grasp the conductor in your right hand with thumb pointing in the direction of current then curling fingers represents the direction of magnetic field".

Short Cut Method

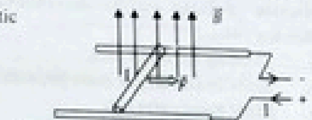
1. کرنٹ کی طرف اپنا Thumb کریں۔
2. Thumb کی side سے دیکھیں تو anti-clockwise direction ہو گی۔
3. دوسری side سے clockwise ہو گی۔

Current *I* is out of the plane of paper	Current *I* is out of the plane of paper
From above: Field is anti-clockwise **From below:** Field is clockwise 	**From above:** Field is Clockwise **From below:** Field is Anti- clockwise 

When two current carrying wires are placed near each other.

When current is in same direction	When current is in opposite direction
	
➢ Wires attract each other. ➢ Since direction of force is always from strong field to weak. Hence ➢ Field is strong at X and Z $(B_{net} = B_1 + B_2)$ ➢ Field is weak at Y $(B_{net} = B_1 - B_2)$	➢ Wires repel each other. ➢ Since direction of force is always from strong field to weak. Hence ➢ Field is strong at Y $(B_{net} = B_1 + B_2)$ ➢ Field is weak at X and Z $(B_{net} = B_1 - B_2)$

FORCE ON CURRENT CARRYING CONDUCTOR IN A UNIFORM MAGNETIC FIELD



When a current carrying conductor is placed in magnetic field it will experience force given as

$$F = ILB\sin\theta$$

➢ Force on the conductor is directly proportional to current passing through conductor. $F \propto I$

➢ Force on the conductor is directly proportional to length of conductor inside the magnetic field $F \propto L$

➢ Force on the conductor is directly proportional to external magnetic field. $F \propto B$

➢ Force on the conductor is directly proportional to $\sin\theta$ where 'θ' is angle between magnetic field and direction of current through conductor. $F \propto \sin\theta$

Magnetic Induction:

Magnetic induction is defined as magnetic force acting on one meter length of conductor, placed perpendicular to the magnetic field lines when one ampere current is passing through the conductor.

$$B = \frac{F}{IL\sin\theta}$$

➢ Its SI unit is Tesla ($T = Nm^{-1}A^{-1} = kgm^{o}s^{-2}A^{-1}$)

➢ Its dimensions are $[ML^{o}T^{-2}A^{-1}]$

Tesla:

Magnetic induction is one tesla if one newton force is acting on one meter length of conductor placed perpendicular to magnetic field lines when one ampere current is passing through the conductor.

Vector form:

Magnetic force on current carrying conductor in vector form is given as

$$\vec{F} = I(\vec{L} \times \vec{B})$$

یاد رکھیں جب بھی دو vectors کا cross product لیں گے تو
تیسرا vector دونوں vectors کے perpendicular ہوگا۔

➢ Magnetic force is always perpendicular to length of conductor and magnetic field lines F⊥L and F⊥B.

➢ Magnetic force is maximum when conductor is placed perpendicular to magnetic field lines. $F_{max} = ILB$

➢ Magnetic force is zero or minimum when conductor is placed parallel or anti-parallel to magnetic field lines. $F_{min} = ILB\sin 0^{o} = 0$

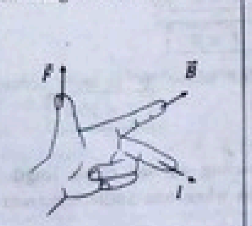
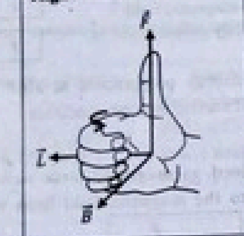
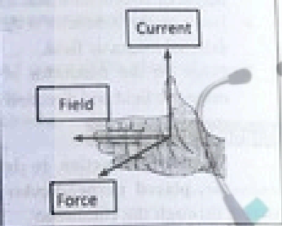
Direction of Force:

Direction of force can be determined following rules.

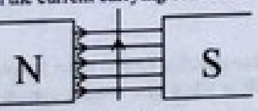
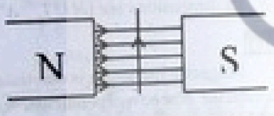
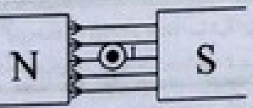
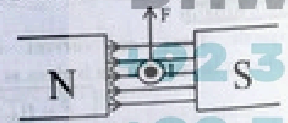
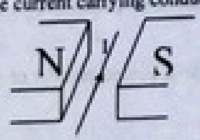
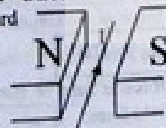
i. ***Fleming left hand rule.***
ii. ***Right hand rule for cross product.***
iii. ***Right hand palm rule***

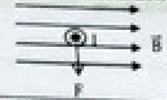
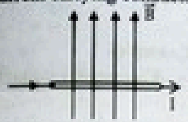
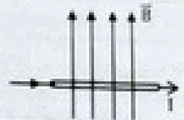
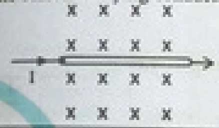
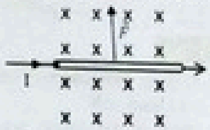
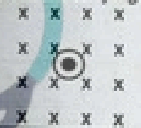
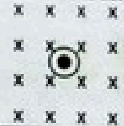
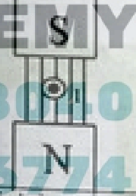
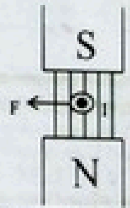
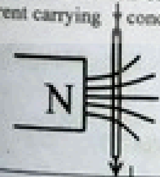
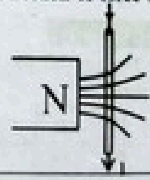
How to Apply Right Hand Palm Rule

1. سب سے پہلے Right hand کو کھولیں۔
2. Fingers کی direction فیلڈ کی طرف set کریں۔
3. ہاتھ کو Rotate کرکے thumb کی direction کرنٹ کی طرف set کریں۔
4. تو جس طرف Palm کی direction ہو گی اس طرف Force حاصل کرے گی۔

Fleming left and rule	Right and rule	Right and palm rule
		

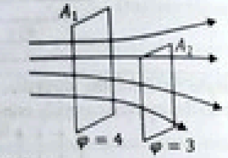
SOME IMPORTANT PRACTICE EXAMPLE FOR RIT AND PALM RULE

Example 1: Find the direction of magnetic force on the current carrying conductor 	**Answer:** Direction of force is into the plane of paper 
Example 2: Find the direction of magnetic force on the current carrying conductor 	**Answer:** Direction of force is upward 
Example 3: Find the direction of magnetic force on the current carrying conductor 	**Answer:** Direction of force is vertically downward 

Example 4: Find the direction of magnetic force on the current carrying conductor (if I is electronic current) 	**Answer:** Direction of force is downward 
Example 5: Find the direction of magnetic force on the current carrying conductor 	**Answer:** Direction of force is out of plane of paper 
Example 6: Find the direction of magnetic force on the current carrying conductor 	**Answer:** Direction of force is upward 
Example 7: Find the direction of magnetic force on the current carrying conductor 	**Answer:** Since current is anti parallel to field line so magnetic force is zero 
Example 8: Find the direction of magnetic force on the current carrying conductor 	**Answer:** Direction of force towrds left 
Example 8: Find the direction of magnetic force on the current carrying conductor 	**Answer:** Direction of force is out of plane of paper 

MAGNETIC FLUX

"Number of magnetic field lines passing through certain area is called magnetic flux through that area."



- Magnetic flux is denoted by ϕ_B and

$$\phi_B = \vec{B}.\vec{A} = BA\cos\theta$$

(Where θ is angle between $\vec{B}$ and vector area)

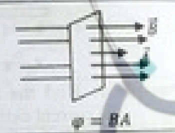
- It is a scalar quantity.
- Its SI unit is Weber $(Wb = T.m^2 = NmA^{-1} = kgm^2s^{-2}A^{-1})$.
- Its dimensions are $[ML^2T^{-2}A^{-1}]$.

Example:

- Flux passing through A_1 is four.
- Flux passing through A_2 is three.

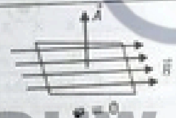
Maximum Flux:

- Magnetic flux is maximum when vector area is parallel to magnetic field lines. OR
- Magnetic flux is maximum when area or plane is held perpendicular to magnetic field lines.



Minimum Flux:

- Flux is minimum when vector area is perpendicular to magnetic field lines. OR
- Flux is minimum when area is held parallel to magnetic field lines.



Short Cut Method

When plane or surface makes an angle 'θ' with magnetic field lines then use the relation.

$$\phi = BA\sin\theta$$

θ	0°	30°	45°	60°	90°
$Cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$Sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Magnetic Flux Density:

Magnetic flux per unit area when area is held perpendicular to magnetic field lines is called magnetic flux density.

$$B = \frac{\phi}{A_{\perp}}$$ ⇒ SI unit is Tesla $(T = Wbm^{-2})$

AMPERE'S LAW

- Ampere's law is stated as

"Sum of all the quantities $(\vec{B}.\Delta\vec{L})$ for all path elements into which complete loop has been divided is equal to μ_o times the total current enclosed by loop"

$$\sum_{i=1}^{n}(\vec{B}.\Delta\vec{L})_i = \mu_o I$$

Where μ_o is permeability of free space and $\mu_o = 4\pi \times 10^{-7}Wbm^{-1}A^{-1}$

- Ampere's law is used to determined magnetic flux density.

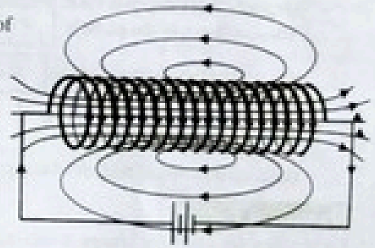
FIELD DUE TO A CURRENT CARRYING SOLENOID

Solenoid:

"Solenoid is a long, tightly wound cylindrical coil which behaves like a bar, magnetic when current passes through it."

Magnetic field outside the solenoid is non-uniform and weak (can be neglected).

- Magnetic field produced at the ends of solenoid is non-uniform and $B_{end} = \frac{\mu_o nI}{2}$ $\left(B_{end} = \frac{B_{center}}{2}\right)$
- Magnetic field produced inside the solenoid strong and nearly uniform.



$$B = \mu_o nI \quad \text{OR} \quad B = \frac{\mu_o NI}{L}$$

Where n is number of turns per unit length.

Magnetic field inside the solenoid depends upon.

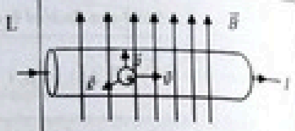
(i). Number of turns of solenoid $(B \propto N)$
(ii). Current passing through solenoid $(B \propto I)$
(iii). Length of solenoid $\left(B \propto \frac{1}{L}\right)$
(iv). Nature of core material (by increasing iron core inside the solenoid magnetic field increases)

Three important cases:

1. If solenoid is stretched then its length increases but no. of turns remains same so magnetic field decreases.
2. If solenoid is compressed then its length decreases but no. of turns remains same so its magnetic field increases
3. If solenoid is cut into two parts and same current passes through each part then magnetic field will remain same because both no. of turns and length become half.

FORCE ON MOVING CHARGE IN A MAGNETIC FIELD

➢ ***Force acting on a current carrying conductor in magnetic field is actually force acting on moving charges.***

No. of charge carriers per unit volume = n No. of charge carriers in volume AL of the conductor = n A L Charge on each particle = q Total charge in the conductor of volume AL = nqAL Time taken by charges to pass through conductor $= \frac{L}{v}$ Current passing through the conductor $= \frac{nqAL}{\frac{L}{v}} = nqAv$	

When charges are moving in the magnetic field they experiences the magnetic force and the resultant of magnetic forces acting on moving charges is the magnetic force acting on current carrying conductor

➢ If a charge q is moving with velocity v in a magnetic field B is given as

$$F = qvB\sin\theta$$ θ is angle between $\vec{v}$ and $\vec{B}$

➢ Magnetic force depends upon

(i). Charge of the particle $F \propto q$

(ii). Velocity of the particle $F \propto v$

(iii). Magnetic field $F \propto B$

(iv). Direction of motion of charged particle. $F \propto \sin\theta$

NOTE: Magnetic force on moving charges is independent of length area or dimensions of the conductor.

Maximum Force:

Force acting on a moving charge is maximum when charge is moving perpendicular to magnetic field lines.

$$F_{max} = qvB\sin 90° = qvB$$

Minimum Force:

Force acting on a particle is zero or minimum when

(i). Charge is zero (neutral particle) $\Rightarrow F = (0)vB\sin\theta = 0$

(ii). Charged particle is at rest ($v = 0$). $\Rightarrow F = q(0)B\sin\theta = 0$

(iii). Magnetic field is zero ($B = 0$) , $\Rightarrow F = qv(0)\sin\theta = 0$

(iv). Charge is moving either parallel or anti-parallel to magnetic field lines.

$F = qvB\sin\theta° = qvB\sin 180° = 0$

Vector Form:

Magnetic force on moving charge in vector form is given as

$$\vec{F} = q(\vec{v} \times \vec{B})$$

➢ Magnetic force is always perpendicular to velocity of charge and magnetic field.

➢ Work done by magnetic force is always zero

(∵ F_m *is perpendicular to velocity and displacement*).

➢ Magnetic force is only deflecting force and it cannot accelerate or decelerate the charge.

➢ When charge in moving in uniform magnetic field its speed, K.E, angular velocity, time period, frequency and angular momentum remain constant.

➢ Angular acceleration and torque by magnetic force is zero.

➢ Only direction of velocity, momentum, acceleration and magnetic force are changing.

Direction of Force:

Direction of force acting on a moving charge in a magnetic field is determined by

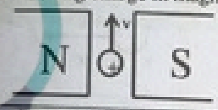
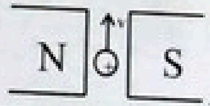
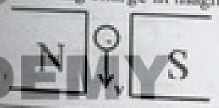
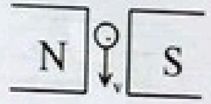
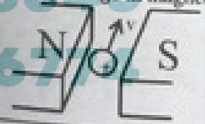
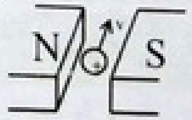
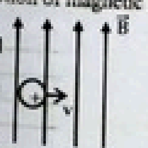
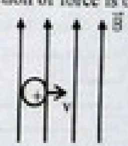
(i). Right hand palm rule

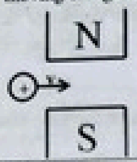
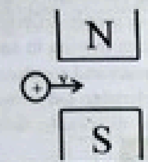
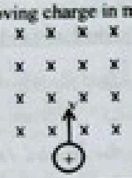
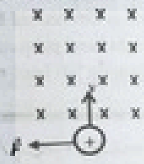
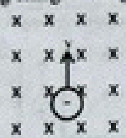
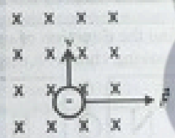
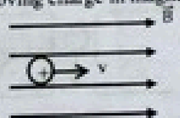
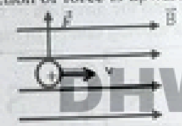
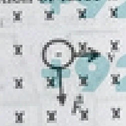
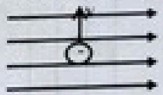
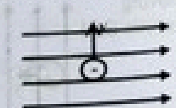
(ii). Fleming left hand rule

یاد رکھیں اگر چارج +ve کی بجائے -ve ہو تو right کی بجائے Left hand palm rule استعمال کریں۔

How to Apply Right Hand Palm Rule

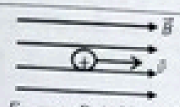
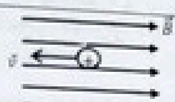
5. سب سے پہلے Right hand کو کھولیں۔
6. Fingers کی direction کو فیلڈ کی طرف set کریں۔
7. ہاتھ کو Rotate کر کے thumb کی direction کو کرنٹ کی طرف set کریں۔
8. تو جس طرف Palm کی direction ہوگی اس طرف Force عمل کرے گی۔

Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is into the plane of paper 
Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is into the plane of paper 
Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is vertically downward 
Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is out of plane of paper 

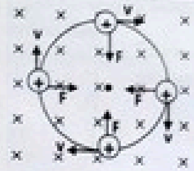
Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is into the of plane of paper 
Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is towards left 
Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is towards right 
Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is upward 
Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is downward 
Example: find the direction of magnetic force acting on a moving charge in magnetic field 	**Answer:** Direction of force is out of plane of paper 

TRAJECTORY OF A CHARGE PARTICLE IN MAGNETIC FIELD

1. Straight Path:

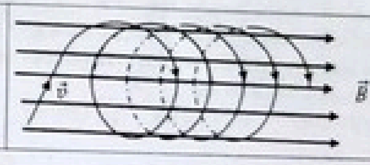
When charge is moving either parallel or anti-parallel to magnetic field lines its trajectory will be straight line.	 $F_m = qvB\sin 0° = 0$	 $F_m = qvB\sin 180° = 0$

2. Circular Path:

When charge is moving perpendicular to magnetic field lines then its trajectory will be circular.		$F_m = F_c$ $qvB = \frac{mv^2}{r}$ $\frac{q}{m} = \frac{v}{Br}$

Radius of circular path	Angular frequency OR Angular Velocity	Time Period	Frequency
$r = \frac{mv}{qB}$	$\omega = \frac{qB}{m}$	$T = \frac{2\pi m}{qB}$	$f = \frac{qB}{2\pi m}$

3. Helical path :

When charge is moving neither parallel, anti-parallel or perpendicular (θ is other than 0°, 90° or 180°) then its trajectory is helical or helix.	

COMPARISON BETWEEN ELECTRIC AND MAGNETIC FORCE

Electric Force	Magnetic Force
1. Electric force is given as $\vec{F}_e = q\vec{E}$.	Magnetic force is given as $\vec{F}_m = q(\vec{V} \times \vec{B})$.
2. Electric force only depends upon charge and electric field and independent of velocity and direction of motion.	Magnetic force depends upon charge, velocity, magnetic field and direction of motion.
3. Electric force is always along the direction of electric field.	Magnetic force is always perpendicular to direction of magnetic field and velocity.
4. Electric force can accelerate, decelerate and deflect the charge.	Magnetic force is only deflecting force and cannot accelerate or decelerate the charge.

LORENTZ FORCE

"If a charge q is moving with velocity $\vec{v}$ in a region where electric field is $\vec{E}$ and magnetic field is $\vec{B}$ then the net force on the charge is vector sum of electric force $q\vec{E}$ and magnetic force $q(\vec{v} \times \vec{B})$."

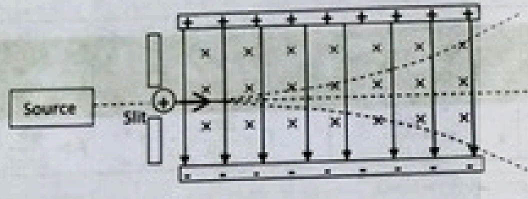
$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

It is known as Lorentz force.

Velocity Selector:

- $\vec{v}, \vec{E}$ and $\vec{B}$ are set mutually perpendicular.
- $\vec{E}$ and $\vec{B}$ are applied in a such a way that they may exert force on moving charge in opposite direction.
- Only these charges passes undeviated for which

$$F_m = F_e$$
$$qvB\sin 90^\circ = qE$$
$$v = \frac{E}{B}$$



Example: Alpha particles ranging in speed from 1000ms^{-1}2000 ms^{-1} enter into a velocity selector where electric field intensity is 300 Vm^{-1} and magnetic induction is 0.20T. The particles which move undeviated will have speed
(a)1000 ms^{-1} (b) 1250 ms^{-1} (c) 1500 ms^{-1} (d) 2000 ms^{-1}

Solution:
$$v = \frac{E}{B}$$
$$v = \frac{300}{0.2} = 1500\ ms^{-1}$$

Example: A velocity selector has magnetic field of 0.30Tand a perpendicular electric field of 10000Vm^{-1}is applied .Then the particles which move undeviated will have speed
(a)330 ms^{-1} (b) 3300 ms^{-1} (c)33000 ms^{-1} (d) 3000 ms^{-1}

Solution:
$$v = \frac{E}{B}$$
$$v = \frac{10000}{0.3} = 33000\ ms^{-1}$$

Example: If a charge q is moving in a velocity selector. The charge will move in a straight path if :
(a)$v = \frac{E}{B}$ (b) E is perpendicular to B.
(c) $F_m = F_e$ (d) All of these✓

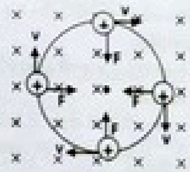
CHARGE TO MASS RATIO OF AN ELECTRON

- Charge to mass ratio $\left(\frac{q}{m}\right)$ of a particle only depends upon nature of particle.
- In case of neutron or any other neutral particle $\frac{q}{m} = 0$ and $\frac{m}{q} = \infty$.
- $\left(\frac{q}{m}\right)_{electron} > \left(\frac{q}{m}\right)_{proton} > \left(\frac{q}{m}\right)_{\alpha-particle}$

Determination of Charge to Mass Ratio of an Electron

- To determined charge to mass ratio of electron beam of electrons is projected in uniform magnetic field in perpendicular direction.
- Magnetic field exerts the force on electrons and bends the beam in a circular path.

$$F_m = F_c$$
$$qvB = \frac{mv^2}{r}$$
$$\frac{q}{m} = \frac{v}{Br}$$



- To determine the radius of circular path beam of electrons is projected in a glass bulb filled with H_2 gas at low pressure due to ionization and de-excitation path of electrons becomes visible.
- If beam of electrons is accelerated through potential difference V then

Gain in K.E:	Gain in Momentum:	Gain in Velocity:
$K.E = qV$	$P = \sqrt{2mqV}$	$v = \sqrt{\frac{2qV}{m}}$

- Putting value of V in eq(i) $\frac{q}{m} = \frac{2V}{B^2r^2}$
- Accurately Known value of e/m for electron is $1.7588 \times 10^{11} Ckg^{-1}$.

Example: A charge particle is moving in a circular path in a perpendicular magnetic field. By increasing the magnetic field charge to mass ratio of the particle will:
(a)Increase (b) decrease (c)Remain same✓ (d) None

Solution:
Charge to mass ratio only depends upon nature of particles.

UNIT 09 ›› ELECTROMAGNETIC INDUCTION

Electromagnetic Induction:

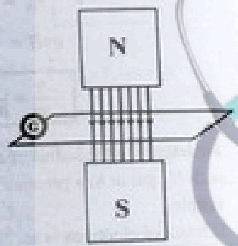
"Changing magnetic flux through a coil induces emf this phenomenon is known as electromagnetic induction."

Induced emf:

Michael Faraday discovered that when magnetic flux linking with conductor changes an emf is produced in the conductor this emf is known as induced emf.

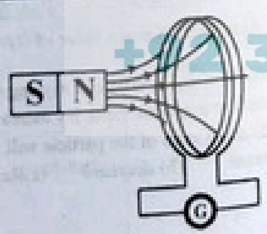
Examples:

1. **Consider a coil placed between the two poles of a magnet.**
   - When coil is stationary no current or emf is induced because magnetic flux is not changing.
   - When coil is moved along the field lines again no emf is induced because magnetic flux is not changing.
   - When coil is moved across (perpendicular) to field lines, magnetic flux changes and emf is induced.

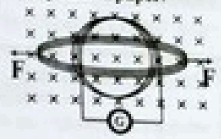


Induced emf and induced current depends upon			
i.	Speed of the coil ($\varepsilon \propto v$).	ii.	Applied magnetic field intensity ($\varepsilon \propto B$).
iii.	Number of turns of the coil ($\varepsilon \propto N$).	iv.	Induced emf is independent of resistance of the coil. But induced current depends resistance ($I \propto \frac{1}{R}$).

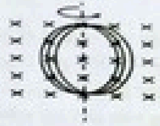
2. **Consider a stationary coil and a bar magnetic is moved near the coil.**
   - When magnet is at rest no emf is induced because magnetic flux is not changing.
   - When magnet is moved towards the coil magnetic flux increases and an emf is induced.
   - When magnet is moved away magnetic flux decreases and an emf is induced.



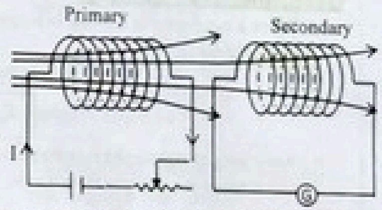
3. **Consider a circular coil placed in a magnetic field directed into the plane of paper.**
   - Since area of the loop is constant hence magnetic flux is constant and no emf is induced in the coil.
   - When coil is distorted its area decreases thus magnetic flux through the coil decrease and an emf is induced.



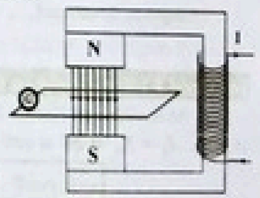
4. **Consider a rotating coil placed in uniform magnetic field directed into plane of paper.**
   - When coil is rotated in magnetic field angle between field lines and vector area changes thus magnetic flux changes and an emf is induced.



5. **Consider a primary coil connected with battery and rheostat and coil connected with a galvanometer is placed near it as secondary.**
   - When current through primary coil is constant magnetic flux through secondary coil is also constant thus no emf is induced in secondary.
   - When current through primary coil increases, magnetic flux through secondary coil increases and emf is induced in secondary coil.
   - When current through primary coil decreases, flux through secondary coil also decreases thus an emf is induced in secondary coil.



6. **Consider coil is placed in the magnetic field of electromagnet**
   - When current through electro-magnet is constant no emf is induced because magnetic flux is not changing.
   - When current through electromagnet increases, magnetic flux increases thus an emf induced due to changing flux.
   - When current through electromagnet decreases, magnetic flux decreases thus an emf is induced due to changing flux.



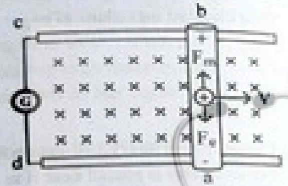
MOTIONAL EMF

"Emf induced by motion of the conductor across the magnetic field is called motional emf."

Dynamic Induced emf:
Emf induced in a conductor when it is moved in stationary magnetic field.

Statically induced emf:
Emf induced in a conductor when conductor is stationary and magnetic field is moving or charge.

Consider a conductor of length L placed on two rails connected with a galvanometer, a uniform. Magnetic field B is applied into plane of paper and rod is moving with uniform velocity v.



Magnetic force on charges:

- When a conductor is moving in magnetic field charges inside the conductor experience magnetic force.

$$F_m = qvB\sin\theta = qvB\sin 90^\circ \quad (\because v \perp B)$$

$$F_m = qvB$$

- According to right hand rule direction of magnetic force on +ve charges is directed upward and it move the charges from a to b.

Electric force on charges:

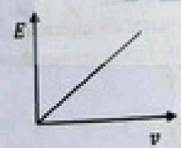
- +ve charges are concentrated at point 'a' and – ve charges are concentrated at point 'b' these +ve and –ve charges induces an electric field E which exerts an electric force on the +ve charges in downward.

$$\vec{F} = q\vec{E}$$

Induced Electric field in Conductor:

At equilibrium electric force will be come equal to magnetic force and net force on charge is zero. $F_e = F_m \Rightarrow qE = qvB$

$$E = vB$$



- Induced electric field depends upon:
  - (i). Speed of conductor ($E \propto v$)
  - (ii). External magnetic field ($E \propto B$)

Induced Potential Difference:

As $\Delta V = -E\Delta r = -vBL$

$$\Delta V = -vBL$$

- Induced electric field depends upon:
  - (i). Speed of conductor ($\Delta V \propto v$)
  - (ii). Length of conductor inside the magnetic field ($\Delta V \propto L$).
  - (iii). External magnetic field. ($\Delta V \propto B$)

Induced emf in the conductor:

induced emf = induced p.d

$$\varepsilon = -vBL$$

Induced Current:

If R is resistance of the current loop abcd then induced current is given as

$$I = \frac{vBL}{R}$$

Induced Charge:

Amount of induced charge in the conductor in a time interval Δt is given as

$$\Delta Q = I\Delta t = \frac{vBL\Delta t}{R}$$

If the angle between velocity of conductor and magnetic field lines is θ instead of 90° then

Induced electric field:

$$E = -vB\sin\theta$$

Induced potential difference:

$$\Delta V = -vBL\sin\theta$$

Induced emf:

$$\varepsilon = -vBL\sin\theta$$

Induced emf depends upon
- (i). Speed of the conductor ($\varepsilon \propto v$)
- (ii). Length of conductor inside the field ($\varepsilon \propto L$)
- (iii). External magnetic field ($\varepsilon \propto B$)
- (iv). Angle between velocity of conductor and magnetic field lines. ($\varepsilon \propto \sin\theta$)

Maximum emf:

Induced emf is maximum when conductor is moving perpendicular to field lines.

$$\varepsilon = vBL\sin 90^\circ$$
$$\varepsilon_{max} = vBL$$

Minimum emf:

Induced emf is minimum when conductor is moving along (parallel or anti-parallel) to field lines.

$$\varepsilon = vBL\sin 0^\circ = 0$$

Example:
An emf of 0.5 V is induced between the ends of a metal bar moving through a magnetic field of 0.20 T. What field strength would be needed to produced an emf of 1.5 V between the ends of the bar. If all other factors remain same.
(a) 0.3 T (b) 0.6 T (c) 0.9 T (d) 1.2 T

Solution:
$(\varepsilon \propto B)$

$$\frac{B_2}{B_1} = \frac{\varepsilon_2}{\varepsilon_1}$$

$$B_2 = \frac{1.5 \times 0.2}{0.5} = 0.6\,T$$

FARADAY LAW

"Average induced emf in a coil of 'N' turns is equal to - ve of rate of change of change of magnetic flux through the coil."

$$\boxed{\varepsilon = -N\frac{\Delta\phi}{\Delta t}}$$

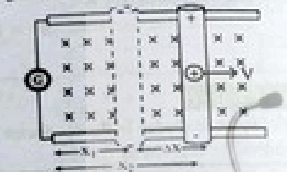
Explanation:
Consider a rod is moving on two rails connected with a galvanometer in presence of magnetic field. As area of loop increases magnetic flux changing through the flux and emf is induced.

$$\varepsilon = -vBL$$

$$\varepsilon = -\frac{\Delta x}{\Delta t}BL$$

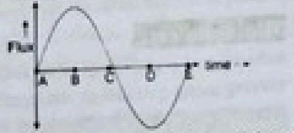
$$\varepsilon = -\frac{\Delta A}{\Delta t}B$$

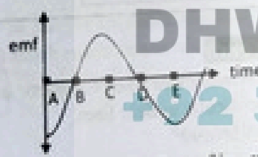
$$\boxed{\varepsilon = -\frac{\Delta\phi}{\Delta t}}$$



- **Induced emf depends upon**
  - (i). Number of turns of the coil ($\varepsilon \propto N$).
  - (ii). Rate of change of flux through the coil $\left(\varepsilon \propto \frac{\Delta\phi}{\Delta t}\right)$.
- Faraday law is used to determine amount of induced emf or induced current.
- Slope of $\phi - t$ graph is directly proportional to induced emf.
- As $slope = \frac{\Delta\phi}{\Delta t} \Rightarrow slope \propto (-\varepsilon)$

Magnetic flux ... change ...
(i) Magnetic ... change ...
(ii) Area change ...
(iii) Magnetic ... Area ... Angle ... change ...





- At instants A and E slope of $\phi - t$ graph is +ve maximum. So emf is - ve maximum.
- At instants B and D slope of $\phi - t$ graph is zero emf also zero.
- At instant C slope of $\phi - t$ graph is - ve maximum. So emf is +ve maximum.

Information
Faraday's designed a Homopolar generator with which he was able to produce continuous induced current.

DIFFERENT FORMS OF FARADAY LAW

1. If a plane is perpendicular to field lines $\phi = BA\,Cos0^\circ = BA$

(i) When $B = constant$ and area is changing then
$\Delta\phi = B\Delta A$ and $\boxed{\varepsilon = N\frac{B\Delta A}{\Delta t}}$

(ii) When $A = constant$ and B is changing then
$\Delta\phi = (\Delta B)A$ and $\boxed{\varepsilon = N\frac{(\Delta B)A}{\Delta t}}$

2. If a plane is not perpendicular to field lines $\phi = BA\,Cos0^\circ = BA$

(i) When $B = constant$ and area is changing then
$\Delta\phi = B\Delta A\,Cos\theta$
and $\boxed{\varepsilon = N\frac{B\Delta A Cos\theta}{\Delta t}}$

(ii) When $A = constant$ and B is changing then
$\Delta\phi = (\Delta B)ACos\theta$
and $\boxed{\varepsilon = N\frac{(\Delta B)ACos\theta}{\Delta t}}$

Induced Current:
If R is resistance of the coil then induced current is given as $I = \frac{\varepsilon}{R}$
or $\boxed{I = N\frac{\Delta\phi}{R\Delta t}}$

Induced current depends upon
- (i). No. of turns of coil (I & N)
- (ii). Rate of change of flux $\left(I \propto \frac{\Delta\phi}{\Delta t}\right)$
- (iii). Resistance of coil $\left(I \propto \frac{1}{R}\right)$

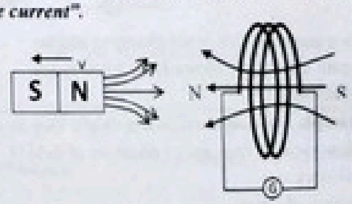
Induced Charge:
Induced charge in a time interval Δt is given as
$\Delta Q = I\Delta t = \frac{N\Delta\phi}{R\Delta t} \times \Delta t$ OR $\boxed{\Delta Q = \frac{N\Delta\phi}{\Delta R}}$

LENZ'S LAW

Lenz pointed out that –ve sign in faraday law $\left(\varepsilon = -N\frac{\Delta\phi}{\Delta t}\right)$ indicates that

"The direction of induced current is always so as the oppose that change which causes the current".

- Whenever magnetic flux changes a current is induced which produces a magnetic field that opposes the change in flux.
  - When flux increases it tends to decreases flux.
  - When flux decreases it tends to increase the flux



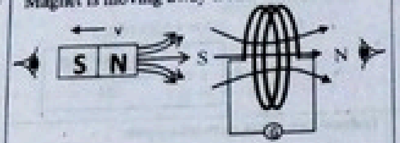
> **NOTE:**
> Lenz's law is in according to law of conservation of energy.

> **NOTE:** Lenz's law is used to determine the polarity of induced emf or direction of induced current.

HOW TO DETERMINE DIRECTION OF INDUCED CURRENT

Case-I When distance between coil and magnetic is increasing or decreasing.

Example 1:
Magnet is moving away from coil



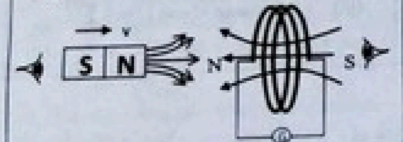
مندرجہ ذیل steps کو ترتیب سے follow کریں:

(i) اگر دور جا رہے ہوں تو ان میں attraction پیدا ہو گی۔

(ii) Magnet والی side پر opposite پول پیدا ہو گا یعنی N والی side پر same پول پیدا ہو گا۔

(iii) N والی side سے دیکھیں تو کرنٹ anticlockwise ہو گا اور 'S' والی side سے current کلاک وائز نظر آئے گا۔

Example 2:
Magnet is moving towards the coil



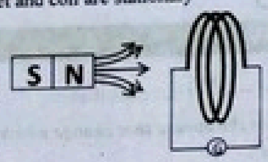
مندرجہ ذیل steps کو ترتیب سے follow کریں:

(i) اگر قریب آ رہے ہوں تو ان میں repulsion پیدا ہو گی۔

(ii) Magnet والی side پر same میگنیٹک پول بنے گا اور دوسری N والی side پر opposite پول پیدا ہو گا۔

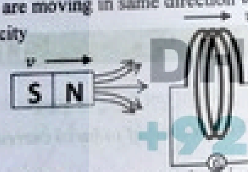
(iii) N والی side سے دیکھیں تو current anticlockwise ہو گا اور 'S' والی side سے current کلاک وائز نظر آئے گا۔

Example 3:
Magnet and coil are stationary



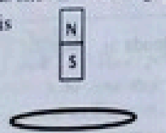
Since magnetic flux is not changing and no current is induced. Hence Lenz's law is not applicable

Example 4:
they are moving in same direction with same velocity

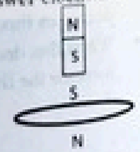


Since magnetic flux is not changing and no current is induced. Hence Lenz's law is not applicable

Example 3: A magnet is falling over a loop as shown in the figure below viewing from above direction of field is
(a) clockwise
(b) anti-clockwise
(c) either clockwise or anti clockwise
(d) no current is induced

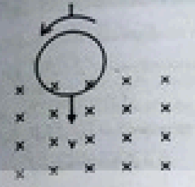


Answer clockwisw



Case-II When magnetic flux is increasing or decreasing:

Example 1: When magnetic flux is increasing:



مندرجہ ذیل steps کو ترتیب سے follow کریں:

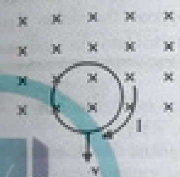
(i) جب کوائل میں entry ہو گی تو اس میں flux بڑھے گا۔

(ii) Induced کرنٹ flux کو کم کرنے کے لیے opposite میگنیٹک فیلڈ پیدا کرے گا۔

(iii) کیونکہ پہلے فیلڈ into the paper ہے تو نیا فیلڈ out of paper پیدا ہو گا۔

(iv) اگر فیلڈ out of paper ہے تو دائیں ہاتھ کی انگلیاں گھمائیں کرنٹ کی direction مل جائے گی۔

Example 2:
When magnetic flux is decreasing:



مندرجہ ذیل steps کو ترتیب سے follow کریں:

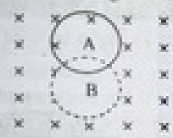
(i) جب کوائل سے exit ہو گی تو اس میں flux کم ہو گا۔

(ii) Induced کرنٹ flux کو زیادہ کرنے کے لیے same direction میں میگنیٹک فیلڈ پیدا کرے گا۔

(iii) کیونکہ پہلے فیلڈ into the paper ہے تو نیا فیلڈ out of paper پیدا ہو گا۔

(iv) اگر فیلڈ پیدا ہونے والے فیلڈ کی direction میں کر کے دائیں ہاتھ کی انگلیاں گھمائیں کرنٹ کی direction مل جائے گی۔

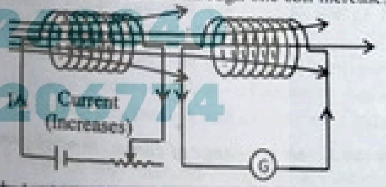
Example 3:
When coil moves from A to B magnetic flux is constant:



Since magnetic flux is not changing hence no current is induced in the coil.

Case-III When current through one coil is increasing or decreasing.

Example 1: When current through one coil increases



Clockwise Current

Induced Current is anti-clockwise

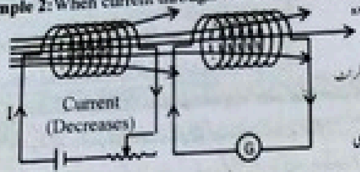
مندرجہ ذیل steps کو ترتیب سے follow کریں:

(i) جب پہلی coil کا کرنٹ بڑھے گا تو دوسری coil کا کرنٹ اس کو کم کرنے کی کوشش کرے گا۔

(ii) اگر پہلی کوائل میں کرنٹ clockwise ہے تو دوسری کوائل میں کرنٹ anti-clockwise پیدا ہو گا۔

(iii) پہلی کوائل میں کرنٹ کی direction دیکھیں اور دوسری کوائل میں کرنٹ کی direction اس کے opposite ہو گی۔

Example 2: When current through one coil decreases.



Clockwise Current Induced Current is clockwise

مندرجہ ذیل steps کو آپ نے follow کرنا ہے
(i) جب پہلی coil کا کرنٹ کم ہو گا تو دوسری coil کا کرنٹ اس کو بڑھانے کی کوشش کرے گا
(ii) اگر پہلی کوائل میں کرنٹ clockwise ہے تو دوسری کوائل میں بھی کرنٹ clockwise ہو گا
(iii) پہلی کوائل میں کرنٹ کی direction جیسی ہوگی دوسری کوائل میں بھی کرنٹ کی direction وہی ہو گی

LEN'Z LAW AND CONSERVATION OF ENERGY

Lenz's law is consistent with law of conservation of energy. whenever current is induced by motion of coil or magnet they experience a magnetic force which opposes the motion of coil or magnet. Thus mechanical energy spent to overcome opposition is converted into electrical energy.

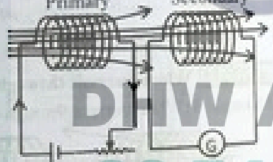
> **NOTE**
> When current is induced in a conductor due to its motion it experiences magnetic force opposite to velocity.

> **NOTE**
> - Whenever magnetic flux increases, magnetic force is repulsive.
> - Whenever magnetic flux decreases, magnetic force is attractive.

MUTUAL INDUCTION

" When current passing through primary coil changes an emf is induced in secondary coil this phenomenon is known as mutual induction".

- Flux $\emptyset$ passing through each turn of secondary coil is directly proportional to current through the primary coil. $N_s\emptyset_s \propto I_p$



$$N_s\emptyset_s = MI_p \quad or \quad N_s\Delta\emptyset = M\Delta I_p$$

- Induced emf in secondary coil is

$$\varepsilon_s = -M\frac{\Delta I_p}{\Delta t}$$

Mutual inductance:

"Ratio of average induced emf in secondary coil to rate of change of current through primary coil is called mutual inductance."

Its SI unit is Henry $(H = VsA^{-1})$

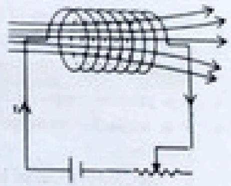
- Mutual inductance depends upon:
  - (i). Number of turns of coils.
  - (ii). Area of the coils.
  - (iii). Closeness and orientation of coils.
  - (iv). Nature of core material.

SELF INDUCTION

"When current passing through the coil changes an 'emf is induced in the coil itself".

Flux passing through each turn of the coil is directly proportional to current passing through coil.

$$N\emptyset \propto I$$



$$\varepsilon = -L\frac{\Delta I}{\Delta t} \Rightarrow \varepsilon \propto L \quad and \quad \varepsilon \propto \frac{\Delta I}{\Delta t}$$

$$N\emptyset = LI \quad and \quad N\Delta\emptyset = L\Delta I$$

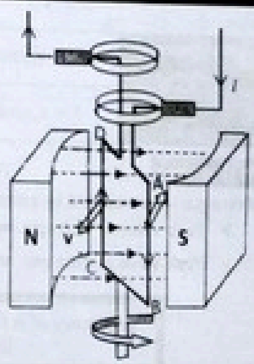
L is proportionality constant and it is known as self inductance.

Self Inductance	Self inductance depends upon
Ratio of average induced emf in the coil to rate of change of current through the coil.	(i). Shape and number of turns of coil
➢ Its SI unit is henry $(H = VsA^{-1})$	(ii). Nature of core material
	(iii). Area and length of the coil

> **Information**
> Induction heater operates on the principle of electromagnetic induction. The water placed on it in the metal pot is boiling whereas that in the glass pot is not. Even the glass top of the heater remain cool to touch. The coil just beneath the top carries ac that produces changing magnetic flux. Flux linking with pots induce emf in them. Current is generated only in the metal pot that heats up the water.

A.C GENERATOR

"A device which converts mechanical energy into electrical energy is called generator."

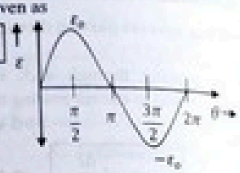


- Generator works on the principle of Faraday law of induction. When coil rotates in magnetic flux changes through coil and an emf is induced in the coil.
- Emf is only induced along the conductor AB and CD because force acting on the charges is along the wire.
- Emf induced along the conductor BC and DA is zero because force on charges is not along the wire

Induced Emf:

Emf induced in generator is dynamically induced emf and it is given as

$\varepsilon = N\omega AB\sin\theta$ or $\varepsilon = \varepsilon_o \sin\theta$



- N is number of turns of the coil.
- ω is angular velocity with which coil is rotated.
- A is area of the coil.
- B is external magnetic field in which coil is rotated.
- θ is angle between velocity of coil and magnetic field lines.
  - Maximum induced emf is given as

$\varepsilon = N\omega AB$ or $\varepsilon_o = N(2\pi f)AB$ or $\varepsilon_o = N\left(\frac{2\pi}{T}\right)AB$

Induced Current:

Induced current is given as $I = \frac{\varepsilon}{R} = \frac{\varepsilon_o \sin\theta}{R}$

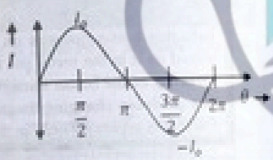
or $I = I_o \sin\theta$

> Current and voltage produced by A.C generator are continuously changing with time.

Where $I_o = \frac{\varepsilon_o}{R} = \frac{N\omega AB}{R}$

In one revolution of generator one cycle of A.C

- Current reverses its direction once.
- Current reaches to maximum value twice.
- Current reaches to zero value twice.
- Emf or voltage reverses its polarity once.



Instantaneous value:

Value of alternating current or voltage at any particular instant of time is called instantaneous value.

Instantaneous voltage	$V = V_o\sin\theta$	$V = V_o\sin\omega t$	$V = V_o\sin 2\pi f t$	$V = V_o\sin\frac{2\pi}{T}t$
Instantaneous current	$I = I_o\sin\theta$	$I = I_o\sin\omega t$	$I = I_o\sin 2\pi f t$	$I = I_o\sin\frac{2\pi}{T}t$

Peak value:

Maximum value reached by current or voltage in a cycle is called peak value.

- Peak value of voltage $= V_o = N\omega AB$
- Peak value of current $= I_o = \frac{V_o}{R} = \frac{N\omega AB}{R}$

> Frequency of A.C used in Pakistan is 50 Hz.

Peak to peak value:

Sum of +ve peak value and −ve peak value is called peak to peak value.

- $V_{p-p} = V_o + V_o = 2V_o$
- $I_{p-p} = I_o + I_o = 2I_o$

Root mean square value:

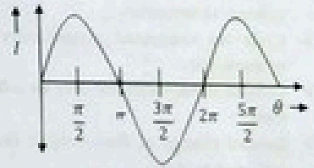
Effective value of alternating current or voltage obtained by taking square root of mean square value is called root mean square value.

For voltage	For current
$V_{rms} = \frac{V_o}{\sqrt{2}} = 0.7V_o$ OR $V_o = \sqrt{2}\,V_{rms} = 1.4V_{rms}$	$I_{rms} = \frac{I_o}{\sqrt{2}} = 0.7I_o$ OR $I_o = \sqrt{2}\,I_{rms} = 1.4I_{rms}$

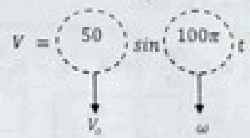
Phase:

Angle θ which specifies the instantaneous value of alternating current or voltage is called phase.

- When $\theta = n\pi$ current is zero
- When $\theta = \frac{odd\pi}{2}$ current is maximum.



Example: If the expression for alternating voltage is $V = 50\sin 100\pi t$. Then by comparing with the standard equation $V = V_o\sin\omega t$ we can find the following as



Peak value	Peak to peak value	rms value	Angular frequency	Frequency	Time period
$V_o = 50V$	$V_{p-p} = 2V_o$ $= 100V$	$V_{rms} = \frac{V_o}{\sqrt{2}}$ $= 0.7 \times 50$ $= 35V$	$\omega = 100\pi$	$f = \frac{\omega}{2\pi}$ $= 50Hz$	$T = \frac{1}{50}$

Example:

If alternating voltage is given as $V = 350\sin 100\pi t$ then rms value of voltage will be

(a) 175V
(b) 700V
(c) 350V
(d) 240V

Solution :

$V = 350\sin 100\pi t$

$V_o = 350V$

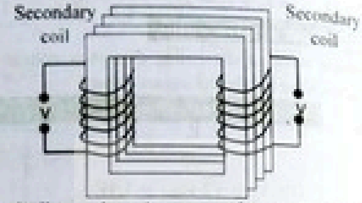
$V_{rms} = \frac{V_o}{\sqrt{2}} = 0.7 \times 350$

$= 240V$

TRANSFORMER

"Transformer is a device which is used to change a given alternating emf into larger or smaller alternating emf".

- Transformer works on principle of mutual induction. Alternating current passing through primary creates a continuously changing magnetic flux through secondary that induces an emf in secondary.



- Transformer only works on A.C and never on D.C.
- Coils of transformer are not connected electrically so there is no transfer of charge from primary to secondary.
- Coils are connected magnetically and power is transferred from primary to secondary magnetically.
- Coils of transformer are over an soft iron laminated core which concentrates the magnetic field lines.
- Rate of change of flux through each cell must be same.

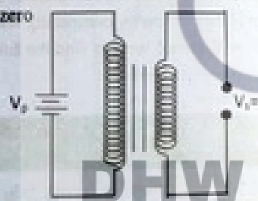
Then

$$V_p = N_p \frac{\Delta\phi}{\Delta t} \quad and \quad V_s = N_s \frac{\Delta\phi}{\Delta t}$$

OR $\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$

Where $\frac{N_s}{N_p}$ is known as transformation ratio.

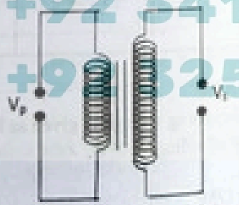
Note:
When a D.C source (battery) is connected with primary then output voltage and power are always zero



Step-up transformer:

A transformer which is used to change a given alternating emf into larger alternating emf.

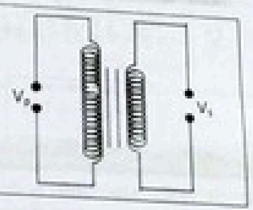
- Increases the voltage level ($V_s > V_p$)
- Decreases the current level ($I_s > V_p$)
- Power level remains same ($P_{in} = P_{out}$)
- Time period or frequency of A.C remains same.



Step-down transformer:

A transformer which is used to change a given alternating emf into smaller alternating emf.

- $N_s < N_p$
- Decreases the voltage level ($V_s < V_p$)
- Decreases the current level ($I_s > V_p$)
- Power level remains same ($P_{in} = P_{out}$)
- Time period or frequency of A.C remains same.



Ideal transformer:

A transformer in which output power is equal to its input power ($P_{loss} = 0$) is known as ideal transformer.

$$P_{in} = P_{out}$$

$$\boxed{V_p I_p = V_s I_s}$$

OR $\boxed{\frac{V_s}{V_p} = \frac{I_p}{I_s}}$ OR $\boxed{\frac{I_s}{I_p} = \frac{N_p}{N_s}}$

If a primary coil of a transformer is connected to A.C mains then $V_p = Constant$

and $\boxed{I_p \propto P_{out}}$

Use of Transformer in Power Transmission:

- If R is resistance of transmission line then power loss in transmission line due to heating effect is given by I^2R.
- The power loss can be reduced by decreasing current level.
- Step-up transformer is used at power generating stations.
- Power is transmitted at high voltage and at low current and thus power loss in transmission line is considerably reduced.
- At other end a step-down transformer is used to decrease the voltage level and increases the current level.

Efficiency of Transformer:

The ratio of output power to input power is known as efficiency of transformer

$$\boxed{\eta = \frac{P_{out}}{P_{in}} \times 100}$$

- For an ideal transformer $\eta = 100\%$ $(P_{in} = P_{out})$
- For a practical transformer due to power losses in transformer $P_{out} < P_{in}$ and efficiency is less than 100%.

OR $\eta = \frac{V_s I_s}{V_p I_p} \times 100$ OR $\eta = \frac{P_{out}}{P_{out} + P_{loss}}$ OR $\eta = \frac{P_{in} + P_{loss}}{P_{in}} \times 100$

Power loss in Transformer:

1. **Eddy Current Loss:**
   "When a metal conductor is placed in a changing magnetic field a current is induced in metal conductor this current is known as eddy current".
   - Some power is lost in form of heat due to eddy current produced in the core of transformer.
   - To reduce the eddy current loss core should be assembled with laminated plates of iron.
2. **Hysteresis loss:**
   "Energy expended to magnetise and demagnetize the core material in each cycle of A.C is called hysteresis loss".
   - Some power is lost to magnetise and demagnetize the core again and again.
   - To reduce the hysteresis loss soft iron core should be used having narrow hysteresis loop.
3. **Cu loss:**
   - When current passes through primary and secondary of transformer some power is lost due to the resistance of coils ($P = I^2R$).
   - To reduce the Cu loss thick Cu wire is used for winding.
4. **Magnetic Flux Leakage:**
   - Flux passing through primary is not completely linked with secondary.
   - To reduce this loss secondary coil is kept inside the primary coil by using "E shaped" plates of iron core.

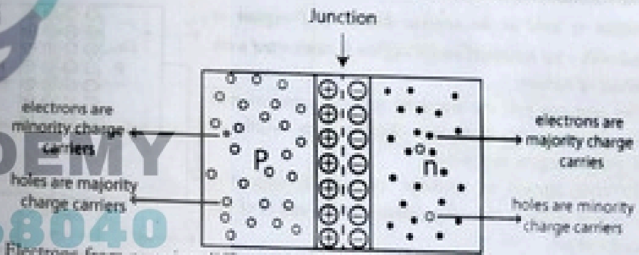
UNIT 10 ›› ELECTRONICS

A review of P-type and n-type semiconductors

P-type semi conductor	n-type semiconductor
Trivalent (III-group) impurity atoms are added in pure semi-conductor.	Penta-valent (V-group) impurity atoms are added in pure semi-conductor.
Trivalent atoms are known as acceptor atoms e.g. B, Al, Ga, etc.	Penta-valent atoms are known as donor atoms e.g. P, Bi, Sb, Sn etc.
Majority charge carriers are holes.	Majority charge carriers are electrons.
Minority charge carriers are electron.	Minority charge carriers are holes.
Holes move from high potential to low potential.	Electrons move from low potential to high potential.
Mobility of holes is low	Mobility of electrons is high.

ELECTRONICS

"When a crystal of Si or Ge is grown in such a way that one half is doped with trivalent impurity and other half is doped with pentavalent impurity p-n junction is formed."



- Electrons from n-region diffuse to p-region thus a layer of +ve immobile ions is formed in p-and n-region and around the junction.

Depletion Region:

"Region produced across the junction containing immobile ions and no charge carriers is called depletion region."

- Depletion region contain immobile (stationary) ions which do not act as charge carriers.

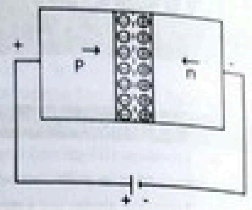
Potential barriers OR Knee Voltage:

"Potential difference produced across the junction which stops the diffusion of electrons is called potential barrier or knee voltage."

- Potential barriers is 0.3V for germanium and 0.7V for silicon.

Forward biases P-n junction

"p-n junction is said to be forward biased if P-region is connected with +ve and n-region with –ve terminal of battery."



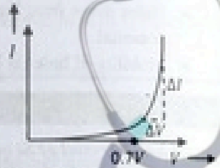
- +ve terminal repel the holes towards the junction and –ve terminal repel the electrons towards the junction thus width of depletion region decreases.
- When applied voltage is greater than potential barrier electrons and holes crosses the junction and a current starts flowing through P-n junction.
- Forward current is due to majority charge carriers and is order of mA.
- Maximum current limit for a junction is decided by power.

Forward Resistance:

Resistance of P-n junction when it is forward biased is called forward resistance.

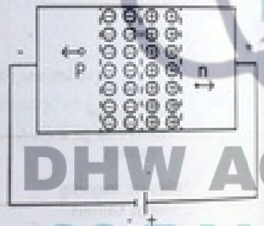
$$r_f = \frac{\Delta V}{\Delta I}$$

(its value is very small in few ohms)

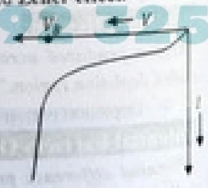


Reverse biased P-n junction:

P-n junction is said to be reverse biased if P-region is connected with – ve terminal and n-region is connected with +ve terminal of battery.



- +ve terminal pull the electrons and –ve terminal pull the holes away from the junction thus width of depletion region increases.
- Reverse current or leakage current through P-n junction is due to minority charge carriers and is of the order of μA.
- If reverse voltage is increased covalent bonds break and large number of electrons are released. This causes a sudden increase in current. This is called **Zener effect.**
- If reverse bias voltage is increased further, minority charge carriers attain high velocity and knock down the bound electrons from covalent bonds by collisions and current increases rapidly this is called **Avalanche effect** or avalanche break down.
- reverse resistance of P-n junction is very large and is of the order of mega-ohms.



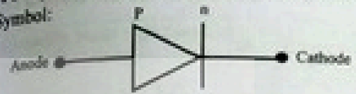
Note: Net current through the junction is due to electrons and holes which is given by

$$I = I_e + I_h$$

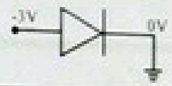
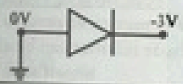
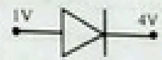
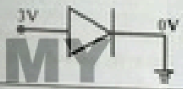
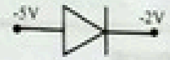
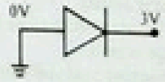
DIODE

A P-n junction is known as semi-conductor diode.

Symbol:



Arrow کرنٹ کی direction کو ظاہر کرتی ہے مطلب Arrow والی direction سے کرنٹ گزر سکتا ہے اور دوسری direction سے کرنٹ نہیں گزر سکتا

	Forward biased diode Examples	Reverse biased diode Examples
	Forward biased کرنے کے لیے anode کا پوٹینشل high اور cathode کا پوٹینشل low ہونا چاہیے	Reverse biased کرنے کے لیے anode کا پوٹینشل low اور cathode کا پوٹینشل high ہونا چاہیے
1.		
2.		
3.		
4.		

RECTIFICATION

"Process of conversion of alternating current into direct current is called rectification."

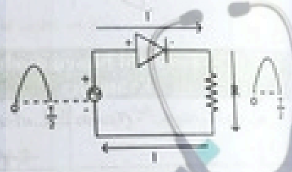
- Device which is used to convert A.C into D.C is called rectifier.
- Diode can be used as rectifier.
- There are two types of rectification.
  i. Half wave rectification ii. Full wave rectification

Half wave rectification:

- ***"A type of rectification in which only one half of A.C is converted into D.C is called half wave rectification."***
- Minimum one diode is required for half wave rectification.

During +ve half cycle:

- Diode is forward biased.
- Resistance of diode will become very small.
- Output pulse is +ve.
- Voltage drop across the diode is approximately zero.
- Voltage drop across the load resistance R is equal to source voltage.



یاد رکھیں

i. +ve half دیتے وقت A.C source کا جو Terminal +ve ہو گا۔

ii. اگر R میں کرنٹ downward ہے تو output pulse +ve ہو گی۔

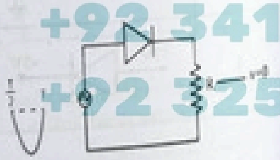
Note

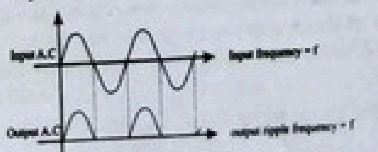
Peak Inverse Voltage: (PIV)

PIV is maximum reverse voltage that a diode rectifier can block in reverse biased state.

During –ve half cycle:

- Diode is reverse biased.
- Resistance of diode will become very large.
- Output voltage is zero.
- Voltage drop across load resistance R is zero.
- Voltage drop across the diode is maximum equal to source voltage.

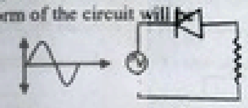




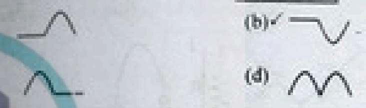
- If 'f' is frequency of input signal then output ripples also have frequency 'f'
- If 'T' is time period of input signal then output ripples also have time period 'T'

Average output in a cycle	Output rms value	Form factor $\left(\frac{V_{rms}}{V_{dc}}\right)$	Ripple Factor $\left(\sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1}\right)$	Efficiency
$V_{dc} = \frac{V_o}{\pi}$	$V_{rms} = \frac{V_o}{2}$	$\frac{\pi}{2} = 1.57$	1.21	40.6%

Example: Input signal applied to the circuit is shown in the figure. The output waveform of the circuit will be

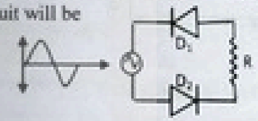


(a) (b)✓ (c) (d)



Solution: -
During positive half diode will become reverse biased.
During negative diode will become forward biased and output will be negative pulse

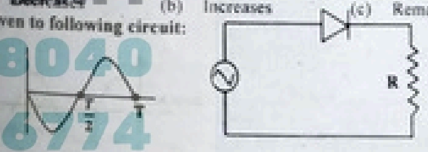
Example: Input signal applied to the circuit is sown in the figure. The output of the circuit will be



(a) Half wave D.C (b) Full wave D.C
(c) A.C (d) Zero✓

Solution:
During positive half diode D_1 will become reverse biased.
During negative diode D_2 will become reverse biased and output will be zero in both halves

1. **The width of depletion region during forward biased mode of a PN-junction diode:**
   [redacted] Decreases (b) Increases (c) Remains same (d) None of these
2. **If is given to following circuit:**



   The output voltage during $0 \to \frac{T}{2}$ will be:
   (a) Positive half (b) Negative half
   [redacted] Zero (d) A.C
3. **A full wave rectifier is being used to rectify an A.C voltage of 110 V, 60 Hz. The number of pulses of rectified current obtained in five seconds is:**
   (a) 300 (b) 60 [redacted] 600 (d) 120
4. **In a full-wave center tap transformer rectifier, how many diodes conduct at a time?**
   [redacted] 1 (b) 2 (c) 3 (d) 4

Full Wave Rectification:

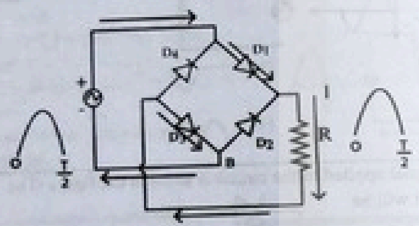
- A type of rectification in which both halves of A.C are converted into D.C is called full wave rectification.
- There are two types of full wave rectification.
  i. Full wave bridge rectifier. ii. Center tap transformer full wave rectifier

Bridge Rectifier:

- Minimum four diodes are used for bridge rectifier.

During +ve half cycle:

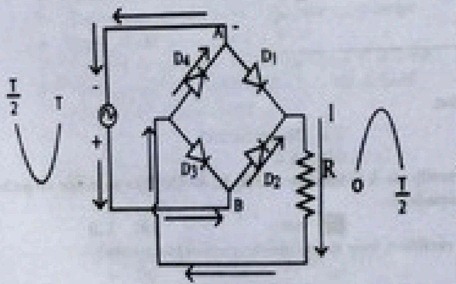
- Point A becomes +ve which makes D_1 forward biased and D_4 reverse biased.
- Point B becomes −ve which makes D_3 forward biased and dereverse biased.
- Diodes D_1 and D_3 conducts but diodes D_2 and D_4 do not conduct the current.

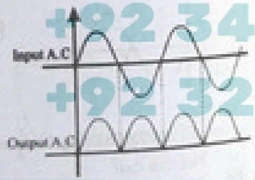


- Maximum voltage is dropped across the load resistance R.

During −ve half cycle:

- Point A becomes which −ve which makes D_4 forward biased and D_1 reverse biased.
- Point B becomes +ve which makes D_2 forward biased and D_3 reverse biased.
- Diodes D_2 and D_4 conduct but diodes D_1 and D_3 do not conduct the current.
- Output is +ve pulse.



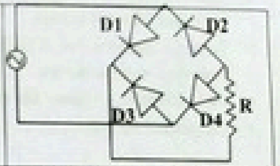


- If 'f' is frequency of input signal then output ripples have frequency '2f'
- If 'T' is time period of input signal then output ripples have time period 'T/2'

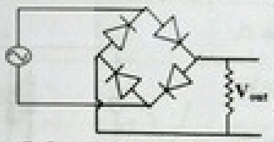
Average output in a cycle	Output *rms* value	Form factor $\left(\frac{V_{rms}}{V_{dc}}\right)$	Ripple Factor $\left(\sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1}\right)$	Efficiency
$V_{dc} = \frac{2V_o}{\pi}$	$V_{rms} = \frac{V_o}{\sqrt{2}}$	1.11	0.48	81.2%

Example:
In the following figure what happens for the positive half cycle of the input?

(a) D_1 and D_4 conduct (b) D_1 and D_2 conduct
(c) D_3 and D_2 conduct (d) D_4 and D_3 conduct



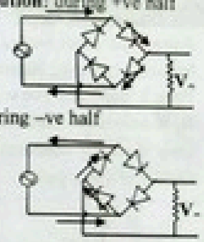
Example: The output of the following circuit will be:



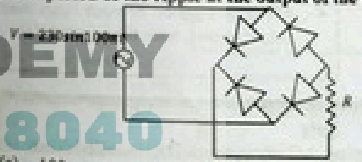
(a) Pulsating full wave D.C (b) Pulsating half wave D.C
(c) Sinusoidal A.C (d) zero✓
In both halves current through resistance is zero

Solution: during +ve half

During −ve half

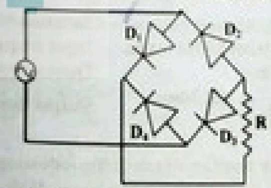


1. **The time period of the ripple at the output of the following circuit is:**



(a) 100 ms (b) 20 ms (c) 50 ms [redacted] 10 ms

2. **In the following figure what happens for the negative half cycle of the input?**



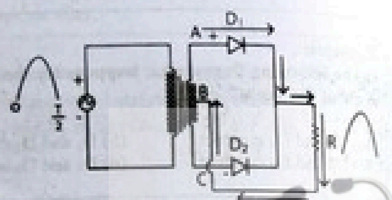
(a) D_1 and D_3 conduct [redacted] D_4 and D_2 conduct
(c) D_3 and D_2 conduct (d) D_4 and D_3 conduct

CENTER TAPE TRANSFORMER RECTIFIER

A center-tap transformer rectifier uses transformer with center-tapped secondary winding which splits the secondary voltage into two parts and two diodes which conduct alternatively.

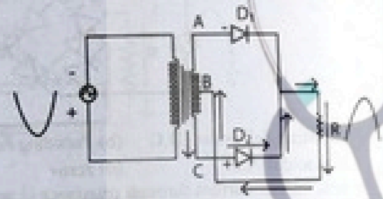
During +ve half cycle:

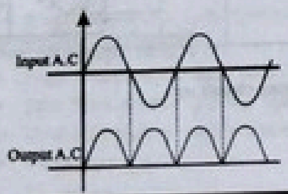
- Point A becomes +ve w.r.t B and diode D_1 becomes forward biased.
- Point C becomes –ve w.r.t B and diode D_2 becomes reverse biased.
- Only diode D_1 conduct the current.
- Output is +ve pulse.



During –ve half cycle:

- Point A becomes –v w.r.t B and diode D_1 becomes reversed biased.
- Point C becomes +ve w.r.t. B and diode D_2 becomes forward biased.
- Only diode D_2 conducts the currents.
- Output is a +ve pulse.





- If 'f' is frequency of input signal then output ripples have frequency '2f'
- If 'T' is time period of input signal then output ripples have time period 'T/2'

Note

- A circuit which converts pulsating D.C into smooth D.C is called filter.
- Capacitor, inductor or their combination can be used as filter.

Example:

If a full wave rectifier circuit is operating from 50 Hz mains, then the time period of output ripples will be:

(a) 20 ms (b) 40 ms (c) 10ms (d) 30ms

Solution:

Input frequency = 50Hz

Output ripple frequency = 100 Hz

Output time period = $\frac{1}{100}$ = 10ms

Example:

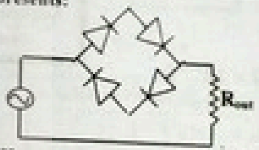
In case of center tape transformer full wave rectifier which of the following statement is true

(a) minimum two diodes are required (b) only one diode conducts at a time

(c) frequency of output ripple is half of input signal (d) all of these ✓

	Half wave rectifier	Full wave bridge rectifier	Center tap transformer rectifier
Minimum of diodes required	1	4	2
Output ripple frequency	f	$2f$	$2f$
Output ripple time period	T	$\frac{T}{2}$	$\frac{T}{2}$
Average output in a cycle	$\frac{V_o}{\pi}$	$\frac{2V_o}{\pi}$	$\frac{2V_o}{\pi}$
Output rms value	$\frac{V_o}{2}$	$\frac{V_o}{\sqrt{2}}$	$\frac{V_o}{\sqrt{2}}$
Form factor	1.57	1.11	1.11
Ripple factor	1.21	0.48	0.48
Efficiency	40.6%	81.2%	81.2%

1. The following circuit represents:



(a) Half wave rectifier (b) Full wave rectifier
(c) Quarter wave rectifier ■ Not a rectifier

2. In a full wave bridge rectifier, how many diodes conduct at a time?
(a) 1 ■ 2 (c) 3 (d) 4

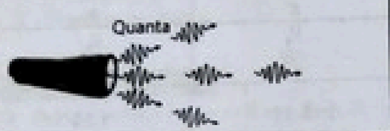
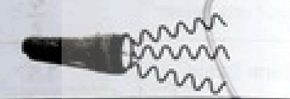
3. If the time period of A.C source applied on the input of full wave rectifier is T_i and time period of the output ripple is T_o, then the relation between these two is:
(a) $T_o = 2T_i$ (b) $T_o = \frac{T_i}{\sqrt{2}}$ (c) $T_o = \sqrt{2}\,T_i$ ■ $T_i = 2T_o$

4. If a full wave rectifier circuit is operating from 50 Hz mains, then the time period of output ripples will be:
■ 10 ms (b) 40 ms (c) 50 ms (d) 80 ms

5. The output voltage of a rectifier is:
(a) Straight line (b) Smooth ■ Pulsating (d) None of these

UNIT 11 ⟩⟩ DAWN OF MODERN PHYSICS

Plank's Assumption:

"Energy is emitted or absorbed by atoms in discrete packets called quanta rather than as a continuous wave."	But according to classical electromagnetic theory of radiations energy was emitted or absorbed by atoms as a continuous wave. And energy is uniformly distributed over the wave.
	Continuous Electromagnetic Wave 

Plank's Law:

Energy of each quantum is directly proportional to its frequency.

$E \propto f$ OR $E = hf$

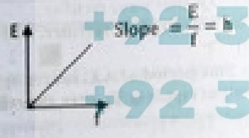
Note
Angular momentum and Plank's constant have same units.

Where 'h' is plank's constant and

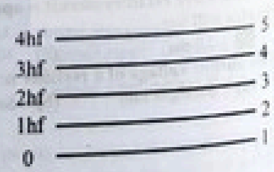
$h = 6.626 \times 10^{-34}\, J.s$ or $h \approx 6.63 \times 10^{-34} J.s$

- SI unit of Plank's constant is $J.s = kgm^2s^{-1}$
- Dimensions of plank's constant are $[ML^2T^{-1}]$

The graph between energy and frequency of photons is a straight line and its slope represent the Plank's constant.



- Atoms or molecules emit or absorb energy when they jump from one quantum state to another.
  (Emitted or absorbed energy = difference in energy between two levels).

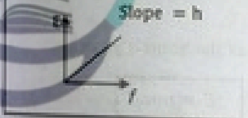
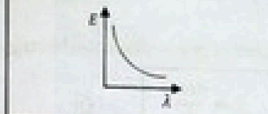
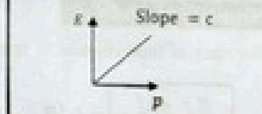


PHOTON THEORY

"According to Einstein photons (discrete energy packets) are integral part of all the electromagnetic radiations."

- These photons carriers energy and momentum.
- Photon cannot be subdivided (elementary particle).
- Rest mass of photon is zero.
- Charge on the photon is zero. Hence they are not affected by electric and magnetic field.
- γ-radiation with energy about $1MeV$. Their quanta can be easily detected.
- Radio waves with energy about $10^{-10}eV$. Their quanta cannot be detected and wave property of radio waves predominates.

Energy of Photon:

Energy of photon in terms of frequency is given as	Energy of photon in terms of wavelength is given as	Energy of photon in terms of momentum is given as
$E = hf$	$E = \frac{hc}{\lambda}$	$E = pc$
$E \propto f$	$E \propto \frac{1}{\lambda}$	$E \propto p$
		

Short cut formula to determine energy of photon :

$$E = \frac{1240 \times 10^{-9}}{\lambda} eV$$

Example:

Energy of blue light photon having wavelength $\lambda = 400nm$ is

(a) 1.3 eV (b) 3.1 eV✓
(c) 1.2 eV (d) 2.1 e V

Solution: $E = \frac{1240 \times 10^{-9}}{400 \times 10^{-9}} = \frac{12.4}{4} eV = 3.1\, eV$

Example:

Which of the following radiations photon carries the most energy

(a) ultraviolet (b) microwaves
(c) visible light (d) x-rays✓

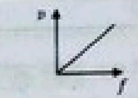
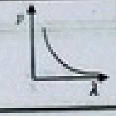
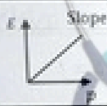
Solution:
x-rays have shortest wavelength
$E \propto \frac{1}{\lambda}$

Example:
Two photons have energies 2 eV and 4eV then the ratio between their wavelength is

(a) 1: 2 (b) 2: 1✓
(c) 1: 1 (d) 1: 4

Solution:
Since $E \propto \frac{1}{\lambda}$
So ratio in wavelength will be inverse ratio of energies

Momentum of Photon:

Momentum of photon in terms of frequency is given as	Momentum of photon in terms of wavelength is given as	Momentum of photon in terms of energy is given as
$p = \frac{hf}{c}$	$p = \frac{h}{\lambda}$	$p = \frac{E}{c}$
$p \propto f$	$p \propto \frac{1}{\lambda}$	$p \propto E$
		

Number of Photons:

If a beam of light contain 'n' number of photons then total energy of the beam is given as

$E = nhf$ Or $E = \frac{nhc}{\lambda}$ Or $E = npc$

Relation with frequency	Relation with wavelength	Relation between momentum
$n = \frac{E}{hf}$	$n = \frac{E\lambda}{hc}$	$n = \frac{E}{pc}$
If two beams have same Energy $n \propto \frac{1}{f}$	If two beams have same Energy $n \propto \lambda$	If two beam have same Energy $n \propto \frac{1}{p}$

- Speed of photons in free space or vacuum is $3 \times 10^8 ms^{-1}$
- Speed of photons in a medium depends upon wavelength

$v \propto \lambda$

ELECTROMAGNETIC WAVES SPECTRUM

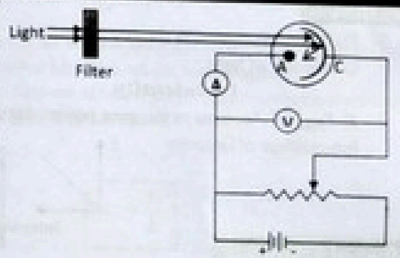
Visible spectrum

- Red has longest wavelength in visible region of spectrum
- Violet has shortest wavelength in visible region of spectrum

PHOTOELECTRIC EFFECT

"Emission of electrons from surface of metal when exposed to light of suitable frequency is called photo-electric effect".

- Emitted electrons are known as photo electrons and current due to photo-electrons is known as photoelectric current.



Stopping Potential:

- By reversing the connections of battery (anode becomes –ve and cathode becomes +ve) electrons are reppelled by anode and photo-electric current decreases.
- **"Negative potential at anode at which photoelectric current becomes zero is called stopping potential."**
- If V_o is stopping potential then maximum K.E of electrons is given as

$K.E_{max} = eV_o$

Example 1: If stopping potential is 0.25V then $K.E_{max} = e(0.25V) = 0.25eV$

$K.E_{max}$ ... stopping potential ... stopping potential

Example 2: If $K.E_{max}$ of electrons is 0.12eV then $V_o = \frac{0.12eV}{e} = 0.12V$

$K.E_{max}$ ... stopping potential ... $K.E_{max}$

Information

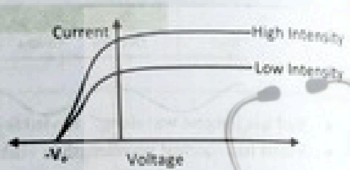
- Photo-electric effect was observed by Heinrich Hertz in 1887.
- Einstein explained the photo-electric effect on the basis of photon theory in 1905.

Note:
Inverse Phenomenon of photoelectric is x-ray production.

Effect of Intensity:

By increasing intensity of light and keeping the frequency (or color)of light constant.

- Photoelectric current increases.
- $K.E_{max}$ or stopping potential remains same.

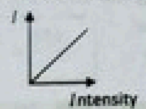


Conclusion:

- Photoelectric current is directly proportional to intensity of light.

$$I \propto Intensity$$

- $K.E_{max}$ of electrons or stopping potential are independent of intensity.



یاد رکھیں

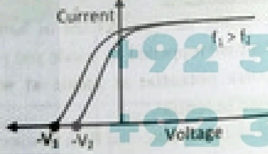
source سے دوری change کرنے سے Intensity change ہوتی ہے
distance change کرنے سے cathode

$$Intensity \propto \frac{1}{(distance)^2}$$

Effect of Frequency:

By increasing frequency of light while keeping the intensity constant.

- Photoelectric current remains same.
- $K.E_{max}$ of electrons or stopping potential increases.



Conclusion:

- Photoelectric current is independent of frequency of light.
- Stopping potential increases by increasing frequency and decreases by decreasing frequency
- $K.E_{max}$ of electrons varies linearly with the frequency.

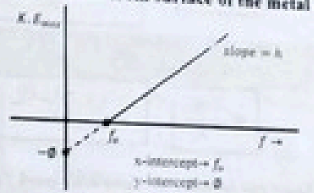
یاد رکھیں

light کی Frequency change کرنے سے colour change ہوتا ہے

Threshold Frequency (f_o):

"Minimum frequency of light required to emit the electrons from surface of the metal is called threshold frequency."

Note:
Threshold frequency only depends upon nature of the metal.



- Below the threshold frequency $K.E$ of electrons will be $-ve$. Hence electrons will not be emitted from the metal (photoelectric effect does not occur) however large the intensity of light may be.
- Electrons are emitted from the metal surface(photoelectric effect does not occur) only when frequency of photon is equal or greater than threshold frequency

Cut off Wavelength:(λ_o)

"Maximum wavelength of light required to emit the electrons from surface of the metal is called cut-off wavelength".

- Electrons are emitted from the metal surface(photoelectric effect does not occur) only when wavelength of photon is equal or less than cut off wavelength.
- If wavelength of photon is greater than cut off wavelength ,electrons are not emitted from the metal surface (photo electric effect does not occur).

Note:
Cut-off wavelength only depends upon nature of the metal.

$$\lambda_c = \frac{c}{f_o}$$

Work function:(ϕ)

"Minimum energy required to emit the electrons from the surface of metal is called work function".

Note:
Work function only depends upon nature of the metal

Metal	Work function (in eV)
Na	2.28
Al	4.08
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14
Fe	4.50

- Electrons are emitted from the metal surface(photoelectric effect does not occur) only when Energy of photon is equal or greater than work function.
- If Energy of photon is less than work function ,electrons are not emitted from the metal surface (photo electric effect does not occur).

➤ Relation between threshold frequency and work function is

$\phi = hf_o$

(اگر f_o اور Ø دونوں میں سے کوئی ایک معلوم ہو تو دوسرے کو relation سے معلوم کر سکتے ہیں)

➤ Relation between cut-off wavelength and work function is

$\phi = \frac{hc}{\lambda_c}$

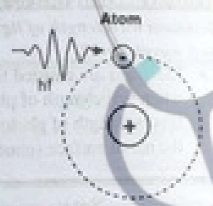
(اگر λ_c اور Ø دونوں میں سے کوئی ایک معلوم ہو تو دوسرے کو relation سے معلوم کر سکتے ہیں)

Short cut formula to determine work function from cut off wavelength OR cut off wavelength from work function is

$$\phi = \frac{1240 \times 10^{-9}}{\lambda_c} eV$$

Einstein Explanation:

➤ According to Einstein beam of light consists of stream of photons and energy of each photon is 'hf' and it only depends upon frequency.



➤ According to Einstein: Intensity of light is directly proportional to number of photons

$$\text{Intensity} \propto \text{no. of photons}$$

★ By increasing intensity number of photons increases thus current increases.

یاد رکھیں

ایک photon صرف ایک electron کو emit کرتا ہے

➤ When a photon strikes an electron it transfers its energy to electron. Some amount of energy is used to remove the electron and rest amount of energy is given to electron as K.E.

➤ Einstein's photo electric effect equation is based on conservation of energy and is given as

$$hf = K.E_{max} + \phi \quad OR \quad K.E_{max} = hf - \phi$$

Where

$$K.E_{max} = eV_o \text{ and } \phi = hf_o = \frac{hc}{\lambda_c}$$

★ By increasing frequency of light $K.E_{max}$ increases thus stopping potential increases and vice versa.

★ By increasing work function or threshold frequency $K.E_{max}$ decreases thus stopping potential decreases and vice versa.

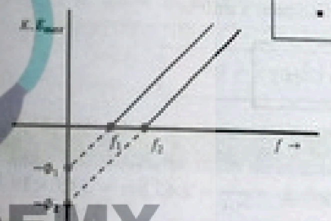
$K.E_{max}$ and V_o increases by	$K.E_{max}$ and V_o decreases by
(i). Increasing the energy or frequency of photon.	(i). Decreasing the energy or frequency of photon.
(ii). Decreasing the wavelength of photon.	(ii). Increasing the wavelength of photon.
(iii). By using a metal having larger work function of threshold frequency.	(iii). By using a metal having smaller work function of threshold frequency.
(iv). By using a metal having smaller cut off wavelength.	(iv). By using a metal having larger cut off wavelength.

For two different metals

➤ Slope of the graph remains same.

➤ $f_2 > f_1$

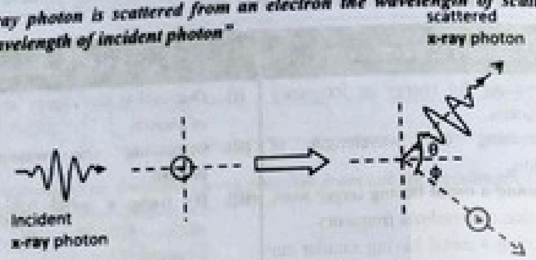
- X-intercept represents the threshold frequency
- Y-intercept represents the work function



Condition	Result	
$f > f_o$ OR $E > \phi$ OR $\lambda < \lambda_c$	Photoelectric effect occur but $I = 0$	$K.E_{max} = 0$ $V_o = 0$
$f = f_o$ OR $E = \phi$ OR $\lambda = \lambda_c$	Photoelectric effect occur but $I \neq 0$	$K.E_{max} \neq 0$
$f = f_o$ OR $E < \phi$ OR $\lambda > \lambda_c$	Photoelectric effect not occur	$K.E_{max} = 0$

COMPTON'S EFFECT

"When an x-ray photon is scattered from an electron the wavelength of scattered photon is greater that wavelength of incident photon"



- When photon strikes with an electron it transfers some amount of its energy and momentum to electron that is why Scattered photon will have less energy, frequency and momentum than incident x-ray photon.(Wavelength of scattered photon is greater than incident photon)
- Compton's effect is best evidence for particle nature of light (photon theory).
- Change in wavelength between scattered and incident photon is called compton's shift.
- By using law of conservation of energy and momentum formula derived for Compton's shift is and momentum formula derived for Compton's shift is

$$\Delta\lambda = \frac{h}{m_o c}(1 - Cos\theta)$$

Compton's Wavelength:

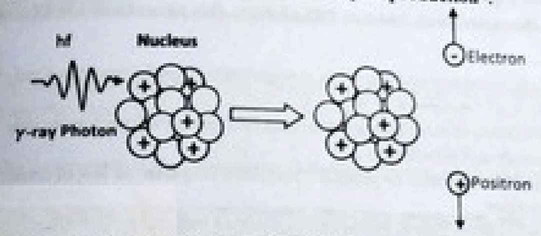
Quantity $\frac{h}{m_o c}$ is known as Compton's wavelength $\lambda_c = \frac{h}{m_o c} = 2.43\ pm = 2.43 \times 10^{-12} m.$

Scattering Angle	Compton's shift in terms of λ_c	Compton's shift in meter
0°	$\Delta\lambda = 0$	$\Delta\lambda = 0$
90°	$\Delta\lambda = \frac{h}{m_o c} = \lambda_c$	$2.43 \times 10^{-12} m$
180°	$\Delta\lambda = 2\left(\frac{h}{m_o c}\right) = 2\lambda_c$	$4.86 \times 10^{-12} m$

- Compton's shift is maximum for $\theta = 180°$

PAIR PRODUCTION

"When a high energy $\gamma - ray$ photon interacts with a heavy nucleus a pair of particle and its anti-particle is produced this phenomenon is known as pair production".



- In pair production a photon (energy) is converted into mass in accordance to Einstein equation $E = mc^2$. $\gamma \rightarrow e^+ + e^-$
- A photon cannot create a single electron or positron alone because it will violate the law of conservation of charge.
- Pair production cannot take place in vacuum (without interaction with nucleus) because it will violate the law of conservation of momentum. To conserve the momentum the pressure of nucleus is required.
- In order to create an electron-positron pair minimum energy $2m_oc^2 = 1.02MeV$ is needed and surpless energy is given to electron and positron as K.E.

$$hf = 2m_oc^2 + (K.E)_{e^-} + (K.E)_{e^+}$$

Minimum energy of photon required	Minimum frequency of photon required	Maximum wavelength of photon required
$E_{min} = 16.38 \times 10^{-14} J = 1.02\ MeV$	$f_{min} = 2.47 \times 10^{20} Hz$	$\lambda_{max} = 1.21 \times 10^{-12} m$

Example:

Pair production can not take place in vacuum because it is against:

(a) law of conservation of energy
(b) law of conservation of charge
(c) law of conservation of momentum✓
(d) all of these

Example:

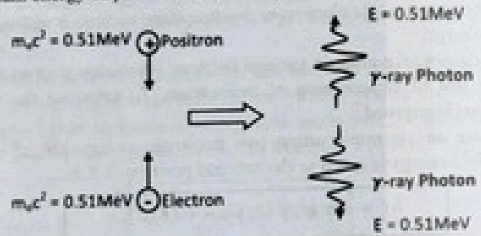
A gamma ray photon having energy 1.04 eV interacts with a heavy nucleus and an electron and positron are produced. The kinetic energy of produced electron will be:

(a) 0.04 eV
(b) 0.02 eV
(c) 0.01 eV ✓
(d) 0.1 eV

ANNIHILATION OF MATTER

When a particle and its anti-particle combine with each other, they destroy each other with the emission of two $\gamma-ray$ photons ,this phenomenon is known as annihilation of matter."

- Mass is converted into energy in accordance to Einstein equation $E=mc^2$.
  $e^- + e^+ \longrightarrow \gamma + \gamma$
- Annihilation of matter always produces two gamma ray photons having same energy, frequency and wavelength .
- Single photon cannot be produced because it is against the law of conservation of momentum
- Single electron or proton cannot be converted into energy because it is against the law of conservation of charge .
- Minimum energy of photon emitted is m_oc^2.



Note:
- Existence of Anti-particle was predicted by Dirac in 1928.
- Positron(Anti-particle of electron) was discovered by Anderson in 1932 from cosmic radiations

Minimum energy of photon emitted	Minimum frequency of photon emitted	Maximum wavelength of photon emitted
$E_{min} = m_oc^2$	$f_{min} = \frac{m_oc^2}{h}$	$\lambda_{max} = \frac{h}{m_oc}$
$E_{min} = 8.19 \times 10^{-14} J = 0.51\ MeV$	$f_{min} = 1.23 \times 10^{20}\ Hz$	$\lambda_{max} = 2.43 \times 10^{-12} m$

UNIT 12 ›› ATOMIC SPECTRA

Spectroscopy:

The study of wavelength and intensity of electromagnetic radiations emitted or absorbed by atoms is called spectroscopy.

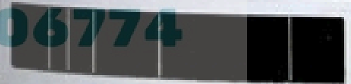
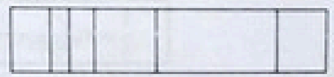
Spectrum:

Set of all the wavelength of electromagnetic radiations emitted or absorbed by a substance is called spectrum

TYPES OF SPECTRUM

1. Line spectrum:

- ***"Spectrum which consists of sharp lines with each line representing a specific wavelength emitted or absorbed by atoms is called line spectrum."***
- Line spectrum is characteristics of emitting elements.
- Line spectrum is due to transition of electrons between energy levels within an atom.
- Each element has a unique set of energy levels, hence each element has a unique line spectrum.
- It is used to identify the gas or element.
- It is also known as atomic spectrum.

Line emission spectrum	Line absorption spectrum
Line spectrum of electromagnetic radiations emitted by a substance is called emission line spectrum	Line spectrum obtained by passing electromagnetic radiations though a substance is called absorption line spectrum
Each line represents the energy or wavelength emitted by the substance	Each line represents the energy or wavelength absorbed by the substance
Shows colored lines with a dark background	Shows dark lines with a bright background
	

Band-spectrum:

"Spectrum which consists of group of lines so closely spaced that each group appears to be a band is called band spectrum or molecular spectrum."

For example: Nitrogen spectrum

- Band spectrum is produced when molecules radiate their rotational and vibrational energies.

2. Continuous-spectrum :

"Spectrum in which there is no gap or space between spectral lines is called continuous spectrum."

- All the solids, liquids and very dense gases when heated produced continuous spectrum.

HYDROGEN EMISSION SPECTRUM

When hydrogen gas is placed in a discharge tube and high voltage is applied across the tube the gas starts glowing and give off bluish red light.

There were five types of series observed in emission spectrum of hydrogen atom

1. Lyman Series:

- Lyman series lies in ultraviolet region of spectrum.
- ***Lyman series is produced when an electron jumps from higher energy level to 1st energy level.***
- The Rydberge formula for Lyman series is

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$ Where $n = 1,2,3, \ldots$

Minimum wavelength of Lyman series is produced when electron jumps from infinite to 1st shell of hydrogen atom. $\lambda_{min} = \frac{1}{R_H} = 91\ nm$	***Maximum wavelength*** of Lyman series is produced when electron jumps from 2nd to 1st shell of hydrogen atom. $\lambda_{max} = \frac{4}{3R_H} = 122\ nm$

2. Balmer Series:

- Balmer series lies in visible region of spectrum.
- ***Balmer series is produced when an electron jumps from higher energy level to 2nd energy level.***
- The Rydberge formula for Balmer series is

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$ Where $n = 2,3,4 \ldots$

Maximum wavelength of Balmer series is produced when electron jumps from 3rd to 2nd shell of hydrogen atom. $\lambda_{min} = \frac{4}{R_H} = 365\ nm$	***Minimum wavelength*** of Balmer series is produced when electron jumps from infinite to 2nd shell of hydrogen atom. $\lambda_{max} = \frac{4}{3R_H} = 656\ nm$

3. Paschan Series:

- Paschan series lies in infrared region of spectrum.
- ***Paschan series is produced when an electron jumps from higher energy level to 3rd energy level.***
- The Rydberge formula for Balmer series is

$$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$ Where $n = 4,5,6 \ldots$

Minimum wavelength of Paschan series is produced when electron jumps from infinite to 3rd shell of hydrogen atom. $\lambda_{min} = \frac{9}{R_H} = 820\ nm$	***Maximum wavelength*** of Balmer series is produced when electron jumps from 3rd to 2nd shell of hydrogen atom. $\lambda_{max} = \frac{144}{7R_H} = 1874\ nm$

4. Bracket Series:

- Bracket series lies in infrared region of spectrum.
- ***Bracket series is produced when an electron jumps from higher energy level to 4th energy level.***
- The Rydberge formula for bracket series is

$$\frac{1}{\lambda} = R_H\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$$ Where $n = 5,6,7 \ldots$

Minimum wavelength of Bracket series is produced when electron jumps from infinite to 4th shell of hydrogen atom. $\lambda_{min} = \frac{16}{R_H} = 1458\ nm$	***Maximum wavelength*** of Bracket series is produced when electron jumps from 5th to 4th shell of hydrogen atom. $\lambda_{max} = 4050\ nm = \frac{400}{9R_H}$

5. Pfund Series:

- Pfund series lies in infrared region of spectrum.
- ***Pfund series is produced when an electron jumps from higher energy level to 5th energy level.***
- The Rydberge formula for Pfund series is

$$\frac{1}{\lambda} = R_H\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$$ Where $n = 6,7,8 \ldots$

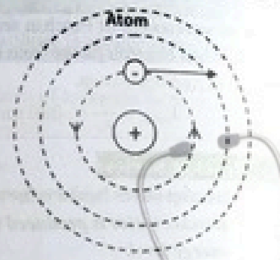
Minimum wavelength of Pfund series is produced when electron jumps from infinite to 5th shell of hydrogen atom. $\lambda_{min} = \frac{25}{R_H} = 2278\ nm$	***Maximum wavelength*** of Pfund series is produced when electron jumps from 6th to 5th shell of hydrogen atom. $\lambda_{max} = \frac{900}{11R_H} = 7455\ nm$

BOHR'S MODEL

- In order to explain emperical results obtained by Rydeberg formulated a model of hydrogen atom.
- Bohr's model is semiclassical model based on following postulates.

According to classical physics an accelerating charge such as orbiting electron must continuously radiate electromagnetic energy.

Postulate I:



"Electron in an atom can move around nucleus in certain circular orbit without radiating. These orbits are called discrete stationary states of the atom."

Bohr's 1st postulate is contradiction of classical physics.

Postulate II:

"Only those circular orbits or stationary states are allowed for which orbital angular momentum is an integral multiple of $\frac{h}{2\pi}$."

$$L = n\left(\frac{h}{2\pi}\right) \quad OR \quad mvr = n\left(\frac{h}{2\pi}\right)$$

Where n is principle Quantum number and $n = 1,2,3 ...$

Example:

What is the ratio between angular momentum of electron in 1st and 3rd shell of hydrogen atom

(a) 1: 3✓ (b) 3:1
(c) 1: 9 (d) 9:1

Example:

In which of the following shell the electron in hydrogen atom will highest angular momentum

(a) K-shell (n=1) (b) L-shell (n=2) (c) M-shell (n=3) (d) N -shell (n=4)✓

Example:

Angular momentum of electron in 3rd shell of hydrogen atom is

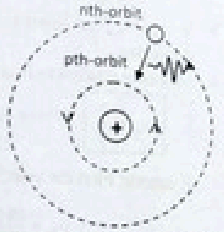
(a) 1.05×10^{-34}J.s (b) 2.10×10^{-34}J.s (c) 3.15×10^{-34}J.s✓ (d) 4.20×10^{-34}J.s

First shell	2nd shell	3rd shell	4th shell	5th shell
$L = \frac{h}{2\pi} = \hbar$	$L = 2\left(\frac{h}{2\pi}\right) = 2\hbar$	$L = 3\left(\frac{h}{2\pi}\right) = 3\hbar$	$L = 4\left(\frac{h}{2\pi}\right) = 4\hbar$	$L = 5\left(\frac{h}{2\pi}\right) = 5\hbar$
$1.05 \times 10^{-34} J.s$	$2.1 \times 10^{-34} J.s$	$3.15 \times 10^{-34} J.s$	$4.2 \times 10^{-34} J.s$	$5.25 \times 10^{-34} J.s$

Postulate III:

"When an electron jumps from high energy state E_n to a low energy state E_p a photon of energy hf is emitted so that,"

$$hf = E_n - E_p$$



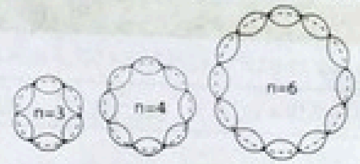
De-Broglie's Interpretation

"According to De-Broglie electron in an orbit behave like a wave and produce stationary wave in the orbit. So length of orbit will be nλ."

$$\ell = n\lambda$$

$$2\pi r = n\left(\frac{h}{mv}\right)$$

$$mvr = n\left(\frac{h}{2\pi}\right)$$



Quantized Radii:

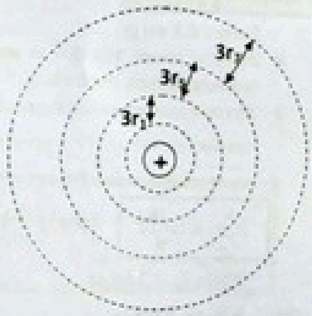
- Electron can move in certain circular orbits.
- Radius of nth shell is given as

$$r_n = \frac{n^2h^2}{4\pi^2kme^2}$$

$$r_n \propto n^2$$

- Short cut relation to find radius

$$r_n = n^2 r_1$$ where $r_1 = 0.053\ nm$.



First shell	2nd shell	3rd shell	4th shell	5th shell
r_1	$r_2 = 4r_1$	$r_3 = 9r_1$	$r_4 = 16r_1$	$r_5 = 25r_1$
0.053 nm	0.212 nm	0.477 nm	0.848 nm	1.325 nm

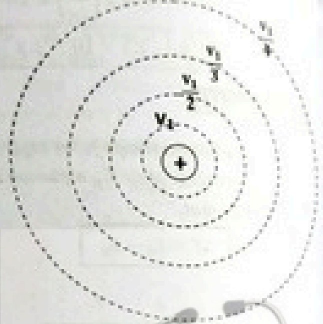
- As 'n' increases radius of orbit increases and distance between two consecutive orbits are also increases.

Quantized Velocity:

Electron moving around the nucleus has discrete velocities.

- Velocity of electron in nth-shell is given as

$$v_n = \frac{2\pi k e^2}{nh} \Rightarrow \boxed{v_n \propto \frac{1}{n}}$$



- In moving from lower to higher shell velocity decreases
- Short-cut relation to find velocity

$$\boxed{v_n = \frac{1}{n}} \quad Where\ v_1 = 2.19 \times 10^6\ m/s$$

Example: Find the velocity of electron in 3rd shell

$$v_3 = \frac{v_1}{3} = \frac{2.19 \times 10^6}{3}\ m/_s = 7 \times 10^5\ m/_s = 7 \times 10^5\ m/_s$$

First shell	2nd shell	3rd shell	4th shell	5th shell
$v_1 = 2.19 \times 10^6$	$v_2 = 1.09 \times 10^6$	$v_3 = 7.3 \times 10^5$	$v_4 = 5.5 \times 10^5$	$v_5 = 4.4 \times 10^5$
$v_1 = 2.19 \times 10^6$	$v_2 = \frac{v_1}{2}$	$v_3 = \frac{v_1}{3}$	$v_4 = \frac{v_1}{4}$	$v_5 = \frac{v_1}{5}$

Quantized Energies:

Note
Orbital electrons have specific amount of energies where as free electron can have any amount of energy.

(i). **Potential Energy:**

- Electron posses P.E due to attraction between electron and nucleus.
- Electron moving around the nucleus has discrete values of P.E given as : $P.E_n = \frac{kq_1q_2}{r_n} = \frac{k(e)(-e)}{r_n} = \frac{-ke^2}{r_n}$
- Short relation to find P.E is

$$\boxed{P.E_n = \frac{-2E_o}{n}} \quad \left(P.E \propto \frac{1}{n^2}\right)$$

Where $E_o = 13.6\ ev$

Note
By increasing n P.E increases.

(ii). **Kinetic Energy:**

- Electron posses kinetic energy due to its motion.
- Electron moving around nucleus has discrete values of K.E.

$$K.E_n = \frac{1}{2}mv_n^2 = \frac{ke^2}{2r_n}$$

- Short cut relation to find K.E is

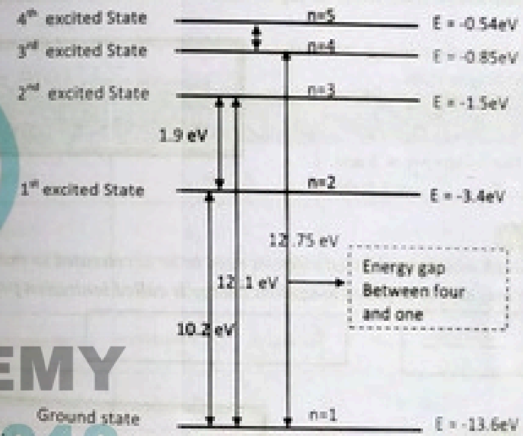
$$\boxed{K.E_n = \frac{E_o}{n^2}} \Rightarrow \boxed{K.E_n \propto \frac{1}{n^2}}$$

(iii). **Total Energy:**

- T.E of electron is given as $E_n = P.E_n + K.E_n = \frac{-ke^2}{r_n} + \frac{ke^2}{2r_n} = \frac{-ke^2}{2r_n} = \frac{-2\pi k^2 me^4}{n^2h^2}$
- Short relation to find energy of electron

$$\boxed{E_n = \frac{-E_o}{n^2}} \quad \left(E_n \propto \frac{-1}{n^2}\right)$$

- Ratio between K.E and total energy of electron is always $1 : -1$
- Ratio between K.E and total energy of electron is always $2 : 1$
- Ratio between K.E and P.E energy of electron is always $1 : -2$



Example 2:
P.E of electron in ground state will be.
(a) 13.6 eV (b) -13.6 eV
(c) -27.2 eV (d) zero eV

Answer:
For ground state $n = 1$ and $E = -13.6eV$
so $P.E = 2(-13.6eV) = -27.2eV$

کسی بھی shell میں الیکٹران کی potential energy اس کی ٹوٹل انرجی سے دو گنا ہوگی۔

Example 1: What is K.E of electron in 1st excited state.
(a) 13.6 eV (b) 3.4 eV
(c) 10.2 eV (d) 12.1 Ev

Answer: For 1st excited state $n = 2$ and $E = -3.4eV$ so $K.E = +3.4\ eV$

کسی بھی shell میں الیکٹران کی K.E اس کی ٹوٹل انرجی کے برابر ہوگی لیکن positive ہوگی۔

First shell	2nd shell	3rd shell	4th shell	5th shell
$-E_o$	$\frac{-E_o}{4}$	$\frac{-E_o}{9}$	$\frac{-E_o}{16}$	$\frac{-E_o}{25}$
$-13.6\ ev$	$-3.4\ ev$	$-1.51\ ev$	$-0.85\ ev$	$-0.54\ ev$

Ionization Energy:

"Energy required to remove the electron from an atom is called ionization energy."

- It is the energy required to make the electron jump from present state to infinite state.
- Ionization energy of electron in nth shell is given as

$$E_{ionization} = \frac{+E_o}{n^2} \quad or \quad E_{ionization} = \frac{13.6\ ev}{n^2} \quad \left(E_{ionization} \propto \frac{1}{n^2}\right)$$

- When an electron jumps from lower to higher orbit 'n' increases and ionization energy decreases.

Example 3: What is ionization energy of electron moving in M-shell of hydrogen atom.

- (a) 13.6 eV (b) 3.4 eV
- (c) 1.5 eV (d) 12.1 eV
- **Answer:** For M-shell $n = 3$ and $E = -1.5\ eV$ so $E_{ionization} = 1.5\ eV$

[illegible] ionization energy [illegible] positive [illegible]

Ionization Potential:

"Potential through which an external electron need to be accelerated so that on collision with bound electron it may supply required ionization energy is called ionization potential."

$$V_{ionization} = \frac{E_{ionization}}{e} \quad or \quad E_{ionization} = eV_{ionization}$$

Example 4:

Minimum potential required to accelerate an external electron so that it may knock out the electron for 1st excited state.

(a) 13.6 V (b) 3.4 V

(c) 1.5 eV (d) 1.5 V

Answer: For 1st excited state $n = 2$ and $E = -3.4\ eV$

so $E_{ionization} = 3.4\ eV$ and $V_{ionization} = 3.4\ V$

[illegible] ionization Potential [illegible] shell [illegible] ionization [illegible] remove [illegible]

Excitation Energy:

- ***"Energy required to make the electron jump from lower state to higher energy state is called excitation energy."***
- Minimum energy required to excite an atom is called excitation energy.

$$E_{excitation} = E_{final} - E_{initial}$$

Example 5: Energy required to excite the hydrogen atom from ground state ($n = 1$) to 2nd excited state ($n = 3$) is

(a) 10.2 eV (b) 1.5 eV

(c) 12.1 eV (d) 3.4 eV

Answer: $E_{exc} = 13.6 - 1.5 = 12.1\ eV$

Excitation Potential:

"Minimum potential through which an external electron need to accelerated so that on collision with bound electron it may supply the required energy is called excitation potential."

$$V_{excitation} = \frac{E_{excitation}}{e}$$

Example 6: Minimum potential required to excite the atom from ground state to first excited state is

(a) 10.2 V (b) 1.5 V

(c) 12.1 V (d) 3.4 V

Quantity	Relation	Electron jumps from lower to higher state	Electron jumps from higher to lower state
Radius	$r_n = n^2 r_1$	Increases	Decreases
Velocity	$V_n = \frac{V_1}{n}$	Decreases	Increases
Momentum	$P_n = \frac{mV_1}{n}$	Decreases	Increases
Angular momentum	$L_n = n\left(\frac{h}{2\pi}\right)$	Increases	Decreases
K.E	$K.E_n = \frac{+E_o}{n^2}$	Decreases	Increases
P.E	$P.E_n = \frac{-2E_o}{n^2}$	Increases	Decreases
Total Energy	$E_n = \frac{-E_o}{n^2}$	Increases	Decreases
Ionization Energy	$I.E_n = \frac{+E_o}{n^2}$	Decreases	Increases
Excitation Energy	$E_{ex} = E_n - E_p$	Decreases	Increases
Time Period	$T_n \propto n^3$	Increases	Decreases

SPECTRAL SERIES

When an electron jumps from high energy state E_n to a low energy state E_p a photon of energy hf is emitted so that,

$$hf = E_n - E_p$$

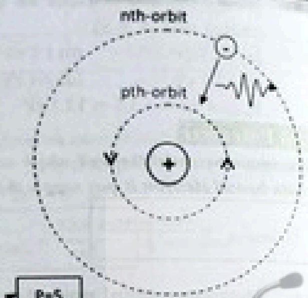
$$\frac{hc}{\lambda} = -\frac{E_o}{n^2} - \left(-\frac{E_o}{p^2}\right)$$

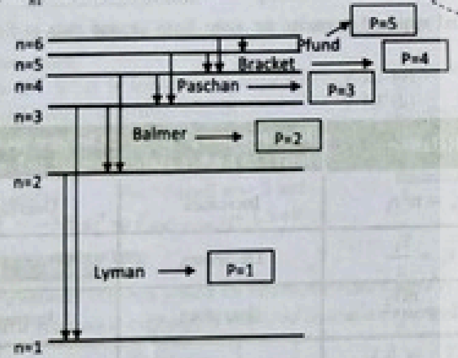
$$\frac{hc}{\lambda} = \frac{E_o}{p^2} - \frac{E_o}{n^2}$$

$$\frac{1}{\lambda} = \frac{E_o}{hc}\left(\frac{E_o}{p^2} - \frac{E_o}{n^2}\right)$$

$$\frac{1}{\lambda} = R_H\left(\frac{E_o}{p^2} - \frac{E_o}{n^2}\right)$$

Where $R_H = \frac{E_o}{hc} = 1.0974 \times 10^7\ m^{-1}$





Series Name	Transition From Higher shell to	Maximum Wavelength	Minimum Wavelength	Region
Lyman	1st shell $P = 1$	$\lambda_{max} = \frac{4}{3R_H} = 122\ nm$	$\lambda_{min} = \frac{1}{R_H} = 91\ nm$	Ultraviolet
Balmer	2nd shell $P = 2$	$\lambda_{max} = \frac{36}{5R_H} = 656.1\ nm$	$\lambda_{min} = \frac{4}{R_H} = 365\ nm$	Visible
Paschan	3rd shell $P = 3$	$\lambda_{max} = \frac{144}{7R_H} = 1874\ nm$	$\lambda_{min} = \frac{9}{R_H} = 820\ nm$	Infrared
Bracket	4th shell $P = 4$	$\lambda_{max} = \frac{400}{9R_H} = 4050\ nm$	$\lambda_{min} = \frac{16}{R_H} = 1458\ nm$	Infrared
Pfund	5th shell $P = 5$	$\lambda_{max} = \frac{900}{11R_H} = 7455\ nm$	$\lambda_{min} = \frac{25}{R_H} = 2278\ nm$	Infrared

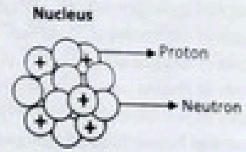
UNIT 13 NUCLEAR PHYSICS

- Rutherford discovered the nucleus and protons.
- Rutherford predicted the existence of neutrons.
- Chadwick discovered the neutrons.

Atomic Nucleus:

"At the center of each atom there is massive and positively charged nucleus containing protons and neutrons."

- About 99.9% mass of an atom is concentrated in the nucleus.
- Size of nucleus is $10^4 - 10^5$ times smaller than atom.
- Volume of nucleus is $10^{12} - 10^{15}$ times smaller than atom.



Unified atomic mass unit:

"One twelfth of mass of carbon-12 is called unified atomic mass unit."

$$1u = \frac{\text{mass of C} - 12}{12} = 1.6606 \times 10^{-27} kg$$

Mass of proton $= 1.673 \times 10^{-27} kg = 1.007276u$

Mass of neutron $= 1.675 \times 10^{-27} kg = 1.008665u$

Mass of electron $= 9.1 \times 10^{-31} kg = 0.00055u$

Atomic Number:

"The number of protons in a nucleus is called atomic number."

- Elements are identified by their atomic number.
- Atomic number is also known as charge number and it identify the charge of nucleus.

$$Charge = Ze$$

For Example atomic number of alpha particle is 2 so its charge will be $(+2e)$.

Mass Number:

"The total number of protons and neutrons in a nucleus is called its mass number".

- Mass number identifies the mass of the nucleus for example mass number of oxygen is 16 so its mass will be 16u.

$$A = Z + N$$

Symbol of Nucleus:

Nucleus is represented by symbol $_ZX^A$.

- Superscript 'A' represents the mass number of total number of nucleus in the nucleus.
- Subscript 'Z' represents the atomic number or total number of protons in the nucleus. For example symbol for uranium is $^{238}_{92}U$

$$No.\,of\ nucleus = 238$$
$$No.\,of\ protons = 92$$
$$No.\,of\ neutrons = 238 - 92 = 146$$

Isotopes:

"Atoms having same atomic number but different mass number are called isotopes."

- Isotopes of hydrogen are $_1H^1, _1H^2, _1H^3$
- Isotopes of carbon are $_6C^{12}, _6C^{13}, _6C^{14}$
- Isotopes of oxygen are $_8O^{16}, _8O^{17}, _8O^{18}$
- Isotopes of Helium are $_2He^3$ and $_2He^4$
- Isotopes of Neon are $_{10}Ne^{20}, _{10}Ne^{21}, _{10}Ne^{22}$
- Cesium and Xenon have 36 isotopes.

	Similarities		Dissimilarities
(i).	Same atomic no.	(i).	Different mass no.
(ii).	Same no. of protons	(ii).	Different no. of neutrons
(iii).	Same no. of electrons	(iii).	Different mass
(iv).	Same chemical properties	(iv).	Different physical
(v).	Same position in periodic table		

Isobars:

"Nuclei having same mass no. but different atomic no. are called isobar."

- $_1H^3$ and $_2He^3$ are isobars.
- $_6C^{14}$ and $_7N^{14}$ are isobars.

یادرکھیں

Mass کے علاوہ باقی کچھ بھی same نہیں ہوگا۔

Isotones:

"Nuclei having same no. of neutrons are called isotones."

- $_6C^{13}$ and $_7N^{14}$ are isotones.
- $_1H^3$ and $_2He^4$ are isotones.

یادرکھیں

Neutrons کی تعداد کے سوا باقی کچھ بھی same نہیں ہوگا۔

Isodiphers:

"Nuclei for which the difference between neutrons and protons is same are called isodiphers."

Example:

$_6H_2^3$ (2 − 1 = 1) and $_6H_7^{13}$ (7 − 6 = 1) are isodiphers.

Mass Spectrograph:

"Mass spectrograph is a device which is used to separate the isotopes and to determine their masses and abundances."

(i). Charged isotopes are accelerated through p.d.V they gain $K.E = qV$ and $V = \sqrt{\frac{2qV}{m}}$

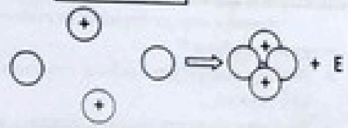
(ii). Then they are projected in magnetic field which exerts a deflecting force which bends in a circular path of different radii depending on their masses. $r \propto \sqrt{m}$

(iii). By measuring their masses by using the relation

$$m = \frac{B^2r^2q}{2V}$$

Mass-defect:

"The difference between total mass of nucleus and experimental mass of the nucleus is called mass defect or mass deficit."



- Loss in mass appears in the form of energy according to Einstein's Equations $E = \Delta mc^2$. Mass-defect is given as

$$\Delta m = Zm_p + (A - Z)m_n - m_{Nucleus}$$

- Mass-defect increases from H to U (mass-defect [illegible] Nucleus [illegible])

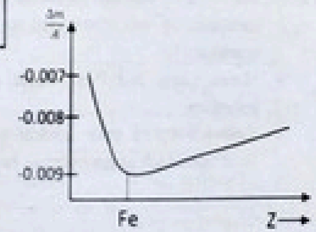
$$\Delta m = 2m_p + 2m_n - m_{He}$$

Mass defect per nucleon: $\left(\frac{\Delta m}{A}\right)$

- Mass-defect per nucleon is also known as packing fraction.
- Mass-defect per nucleon or packing fraction measures the stability of a nucleus.

$$Stability \propto \frac{\Delta m}{A}$$

- $\frac{\Delta m}{A}$ increases from H to Fe
- $\frac{\Delta m}{A}$ is maximum for $_{26}Fe$
- $\frac{\Delta m}{A}$ decreases from Fe to U

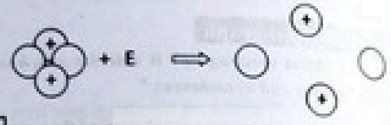


Binding Energy:

"Energy required to break the nucleus into its constituents protons and neutrons is called binding energy."

- Energy appears in increase in mass.
- Binding energy of a nucleus is given as

$$E_B = \Delta mc^2$$



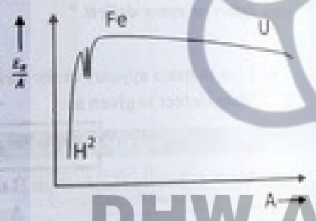
OR

$$E_B = [Zm_p + (A - Z)m_n - m_{Nucleus}]c^2$$

- Binding energy increases from H to U.
  ([illegible] Nucleus [illegible] binding [illegible])

Binding Energy Per Nucleon:

"Average energy required to remove a single nucleon from the nucleus is called binding energy per nucleon."



- Binding energy per nucleon measures the stability of a nucleus.
- Binding energy per nucleon increases from H to Fe.
- Binding energy per nucleon is maximum for $({}_{26}Fe^{56})$ and its value is 8.8 Mev.
- Binding energy per nucleon decreases from Fe to U.
- For U^{238} binding energy per nucleon curve rises, products are more stable than reactants and energy is emitted (e.g. fission and fusion).

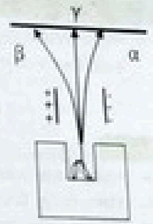
RADIOACTIVITY

"Unstable isotopes or elements emit radiations spontaneously these elements are known as radioactive elements and this phenomenon is known as radioactivity."

- Radioactivity was discovered by Henry Becquaral in 1896 by observing radiations from uranium.
- Marrie Curie and Piere Curie discovered two new radioactive elements radium and polonium.
- Radioactivity is pure nuclear phenomenon and it is independent of other physical conditions such as temperature pressure, electric and magnetic field etc.

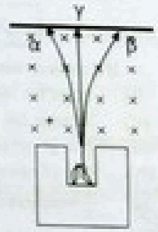
Types of radiations:

In presence of electric field



- Radiations are of three types α, β and γ.
- γ-rays pass straight showing that they are neutral.
- β-rays are deflected towards +ve plate showing that they are negatively charged.
- α-rays are less deflected showing that they are massive and β-rays are more deflected showing that they are lighter.

In presence of magnetic field:



- γ-rays are not deflected by magnetic field because they are neutral.
- α and β rays are deflected in opposite directions because they are oppositely charged.
- α is less deflected than β because α is massive than β.

Properties of rays:

Features	α-rays	β-rays	γ-rays
Nature	Helium Nuclei	Electrons or positrons	E.M. photons
Typical source	Radon-222	Strontium-94	Cobalt-60
Mass No.	$A = 4$	$A = 0$	$A = 0$
Charge No.	$Z = 4$	$Z = -1 \text{ or } +1$	$Z = 0$
Mass	$4u$ or $4m_p$	m_e	Mass less
Charge	$+2e$	$-e$ or $+e$	zero
Speed	$\sim 10^7 m/s$	$\sim 10^8 m/s$	$\sim 3 \times 10^8 m/s$
Penetrating power Range (in air)	Several centimeter	Several meter	Obey inverse square law
Ionizing ability (ions in pair in air per mm)	$\sim 10^4$	$\sim 10^2$	~ 1
Energy spectrum	Line and discrete	Continuous	Line and discrete
Effect of electric or magnetic field	Deflected	Deflected	Not deflected
Absorbed by	A paper	1-5 mm *Al* sheet	1-10 cm of *lead* sheet

NUCLEAR TRANSMUTATION

"Conversion of parent nucleus into a daughter nucleus by emission of radiations is called nuclear transmission or nuclear decay or nuclear disintegration."

$$_ZX^A(\text{parent}) \rightarrow {}_{Z'}Y^{A'}(\text{daughter}) + radiations$$

α-decay:

- Alpha decay occurs with nuclei that are too large to be stable.
- Alpha decay is caused by coulomb repulsion.
- General reaction for α-decay is

$$_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + He_2^4$$

- If a nucleus emits an alpha particle its
  - (i). Mass No. decreases by 4.
  - (ii). Atomic No. decreases by 2.
  - (iii). No. of protons decreases by 2.
  - (iv). No. of neutrons decreases by 2.
  - (v). $\frac{N}{Z}$ ratio increases.

β-decay:

- Beta decay is caused by weak nuclear force.
- There are three types of β-decay
  (i) -ve beta (β^-) (ii) +ve beta (β^+) (iii) electron capture
- General reaction for β^- is

$$_ZX^A \rightarrow {}_{Z+1}Y^A + e_{-1}^0 + \bar{\nu}$$

Note
β^- is due to neutron decay into a proton, electron and anti-neutrino
$$_0^1n \rightarrow {}_1^1H + e_{-1}^0 + \bar{\nu}$$

- If a nucleus emits a β^- particle its
  - (i). Mass No. remain same.
  - (ii). Atomic No. increases by one.
  - (iii). No. of neutrons decreases by one.
  - (iv). No. of protons increase by one

Positive Beta decay:

- General reaction for β^+ is

$$_ZX^A \rightarrow {}_{Z-1}Y^A + e_{+1}^0 + \nu$$

Note
β^+ is due to proton decay into a neutron, positron and anti-neutrino
$$_0^1n \rightarrow {}_1^1H + e_{+1}^0 + \nu$$

- If a nucleus emits a β^+ (positron) its
  - (i). Mass No. remains same.
  - (ii). Atomic No. decreases by one
  - (iii). No. of protons decreases by one.
  - (iv). No. of neutrons increases by one.

Electron capture:

For few nuclei electron usually from K-shell is captured by nucleus

$$_ZX^A + e_{-1}^0 \rightarrow {}_{Z-1}Y^A + \nu$$

Note
Neutrino (ν) and anti-neutrino ($\bar{\nu}$) have zero charge and approximately zero mass.

- If a nucleus captures an electron its
  - (i). Mass No. remains same.
  - (ii). Atomic No. decreases by one
  - (iii). No. of protons decreases by one.
  - (iv). No. of neutrons increases by one.

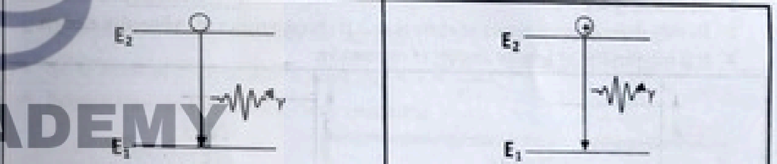
Gamma Decay:

Gamma rays are photons just like ultraviolet and X-rays and only differs on the basis of its origin or energy.

- As there are energy levels for electrons in an atom similarly there are energy levels for nucleons in a nucleus.
- Energy gap between nuclear levels are of order of MeV but in case of atoms energy gaps are only few electron volt.
- When a nucleus is excited (a nucleon jumps from low energy state to higher energy state) it can decay to ground state by emission of γ-ray photon.
- General reaction for γ-decay is

$$_ZX^A \rightarrow {}_ZX^A + \gamma$$

Mass No. and atomic No. of the nucleus remains same.



Neutron jumps from higher to lower state | Proton jumps from higher to lower state

HALF LIFE

"Time in which half no. of radioactive element decay is called its half life"

Random process:

A process without defined pattern, rule or method is called random process. Nuclear decay is random process because it is unpredictable that which atom when will decay.

Spontaneous Process:

A process occurring without apparent external cause is called spontaneous process. Nuclear decay is spontaneous process it cannot be speeded up or slowed down by physical and chemical means.

- Since individual disintegrations are random, however the probability of decay (half life) in a sample has fixed value which is characteristic of that material.

1. ***"No radioactive element can completely decay or infinite time is required for all the atoms to decay".***

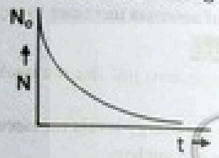
2. **Rate of decay:** According to Rutherford and Soddy law for radioactive decay:

" Rate of decay of an element at any instant is directly proportional to number of atoms present at that instant".

$$\frac{\Delta N}{\Delta t} \propto -N$$

(-ve sign یہ ظاہر کرتی ہے کہ atoms کی تعداد کم ہو رہی ہے۔)

$$\frac{\Delta N}{\Delta t} \propto -\lambda N$$



If N_o are number of atoms at $t = 0$ then number of atoms at any instant t are given as

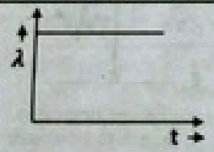
$$N = N_o e^{-\lambda t}$$

Decay Constant (λ):

"Fractional decay per second is known as decay constant or disintegration constant."

$$\lambda = \frac{\Delta N / N}{\Delta t}$$

- Its SI unit is sec^{-1}.
- Its only depends upon nature of element or it is characteristics of radioactive element.
- It is independent of time or amount of radioactive.

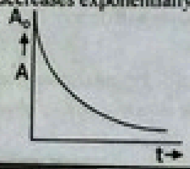


Activity: ***"No. of disintegrations per second is called activity or rate of decay."***

$$A = \lambda N$$

OR $A = A_o e^{-\lambda t}$

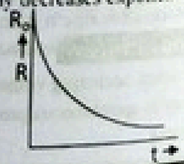
Activity decreases exponentially with time.



$$R = \lambda N$$

OR $R = R_o e^{-\lambda t}$

Rate of decay decreases exponentially with time.



Activity or rate of decay depends upon time, No. of atoms and nature of radioactive element.

- SI unit of activity is Becquaral: $1\,Bq$ = one disintegration per sec
- A common unit of activity is curie:
  $1Ci = 3.7 \times 10^{10} Bq = 3.7 \times 10^{10}$ disintegration per sec

Half life:

- Half life of a radioactive element only depends upon nature of element and independent of time and no. of atoms.
- After each half life no. of atoms will become half of its initial value.

No. of half lives	1	2	3	4	5
Fraction remain undecayed	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$
Fraction decayed	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{31}{32}$

(اوپر والے table کی مدد سے اگر half-life کی تعداد دی ہو تو fraction معلوم کر سکتے ہیں اگر fraction دیا ہو تو half-life کی تعداد معلوم کر سکتے ہیں)

Note
One curie is approximately equal to activity of one gram of radium.

If 't' is total time for 'n' half lives then

$$t = nT_{1/2}$$

(اگر کوئی دو چیزیں معلوم ہوں تو اس فارمولے سے تیسری معلوم کریں)

- No. of atoms of undecayed after nth half life: $N = \frac{N_o}{2^n}$
- No. of atoms of decayed after nth half life: $N = \frac{(2^n - 1)N_o}{2^n}$
- Relation between half life and decay constant is

$$T_{1/2} = \frac{l_n(2)}{\lambda}$$

OR $$T_{1/2} = \frac{0.0693}{\lambda}$$

Short Cut
$$T_{1/2} = \frac{0.7}{\lambda}$$
اگر ان میں سے کوئی ایک معلوم ہو تو اس فارمولے سے دوسری کو معلوم کریں۔

Mean life:

Time in which 63% of a radioactive element is decayed is known as mean life.

$$mean\ life = T^* = \frac{1}{\lambda}$$

- $T^* = \frac{T_{1/2}}{0.693} = 1.44\ T_{1/2}$
- Mean life is about 44% more than half life.

Isotope	Half life	Isotope	Half life
Uranium-238	4.5×10^9 years	Sodium-24	15 hours
Radium-226	1620 years	Iron-59	45 days
Uranium-239	23.5 minutes	Technetium-99	6 hours
Radon	3.8 days	Iodine-125	60 days
Iodine-131	8 days	Plutonium	24000 years

RADIATION EXPOSURE

Back ground radiations:

- *"Radiations present in the environment whose source unknown is are called back ground radiations."*
- Background radiations varies from place to place.
- Source of background radiation are
  - Cosmic radiations
  - Radioactive substance in earth's crest.
  - Building materials containing small amount of radioactive substance.
  - Radioactive radon gas enters buildings from ground.
  - All types of food contain small amount of radioactive substance common are potassium-40 and carbon-14 isotopes.
  - Radiations added in environment by human activities such as medical practices diagnostic x-rays.
  - Other sources include radioactive waste from nuclear facilities, hospitals and research centers, colour T.V, luminous watches and tobacco leaves

Cosmic Radiations:

"Radiations coming from outer space to earth in form of high energy electromagnetic radiations and charged particles are called cosmic rays."

- Atmosphere acts as a shield to absorb some of these radiations.
- Ozone layer absorb ultraviolet radiations which causes eye and skin diseases.
- Recently a depletion in ozone layer was observed due to a chemical chloro-floro carbon which is used in aerosol sprays, refrigeration, paints and foam industry.

BIOLOGICAL EFFECTS OF RADIATIONS

Absorbed dose:

"The energy E absorbed from ionzining radiation per unit mass m of the of absorbing body is called absorbed dose."

$$D = \frac{E}{m}$$

- Its SI unit is gray ($1Gy = 1Jkg^{-1}$).
- Its old unit is "rad" (radiation absorbed dose)
  $1\,rad = 0.01\,Gy$ or $1\,Gy = 100\,rad$
- Equal dose of different radiations don't produce same biological effect.
- Biological effect depends upon two factors
  1. Types of radiation
  2. The part of body absorbing radiation.
- Neutrons are more damaging to eyes than other parts of body.
- For same absorbed dose α-rays are 20 times more damaging than x-rays.

Equivalent Dose:

"The product of absorbed dose and RBE (relative biological effectiveness) of the kind of radiation being absorbed is called equivalent dose."

$$D_e = D \times RBE$$

- It's SI unit is sievert (Sv): $1Sv = 1\,Gy \times RBE$
- It's old unit is "rem" (Rontgen equivalent mass)
  $1\,rem = 0.01\,Sv$ or $1\,Sv = 100\,rem$
- Background radiation to which we expose on average is 2mSv per year.
- Doses of 3Sv will cause radiation burn to the skin weekly dose of 1mSv is consider to be safe for the workers of nuclear facilities or mines.

Dosage in microsievert	Effect	Dosage in microsievert	Effect
1×10^6	Radiation sickness	2.5×10^6	Sterility for about two years
1.5×10^6	Temporary low fertility	4×10^6	Death of 60% of people exposed

- The damage from α-particle is small until it enters the body.
- α and β particles cause redness and sores on the skin.

Low level radiation effects	High level radiation effect
• Loss of hair • Ulceration • Stiffening of lungs • Drop in white blood cells which results in sickness pattern of diarrhea, vomiting and fever known as radiation sickness.	• Disrupt blood cells seriously lead to anemia and leukemia • Chromosome abnormalities or mutation may cause delayed genetic effects such as cancer and eye cataracts and abnormalities in future generation.

BIOLOGICAL AND MEDICAL USES OF RADIATIONS

- Radioisotopes are used to find out what happens in many complex chemical reactions and how they proceed.
- In biological, they have helped in investigating into chemical reactions that take place in plants and animals.
- Radio active isotopes are used to determine proper amount of fertilizer taken up by plant.
- Radiation induced mutation improved varieties of crops. Such as rice, chickpea, wheat and cotton etc.

Radio active Tracer:

"Tracer technique is to substitute radioactive atoms** for stable atoms of same kind and **then follow the radioactive atoms with the help of radiation** detector. These radioactive elements **are known as radioactive tracers."

- In medicine, tracers are used to detect malignant tumors.
- In agriculture, tracers are used to study the uptake of a fertilizer.
- Tracers are used to identify faults in underground pipes.

Radioactive Element	Isotope	Use in medicine
Iodine	I-131	Mostly absorbed in thyroid gland ∵ Hyper active gland absorb more than twice amount of iodine than normal gland • Used for treatment of thyroid gland cancer
Phosphorous	P-32	Mostly absorbed by bones used for treatment of skin cancer.
Cobalt	Co-60	Mostly absorbed by liver used for treatment of cancer.
Sodium	Na-24	It is uniformly disturbed throughout the body and used to study the circulation of blood.
Strontium	Sr-90	Used for treatment of skin cancer.

- The γ-rays radiograph is used in medical diagnosis such as internal imaging of brain and determine precisely the size and location of tumors.

BASIC FORCES OF RADIATIONS

- Basic forces of nature are:
  1. Gravitational force
  2. Magnetic force
  3. Electric force
  4. Weak nuclear force
  5. Strong nuclear force

- Electric and magnetic forces were united to get an electro magnetic force by Faraday and Maxwell.
- In 1979, Glashow Weinberg and Abdus Salam shared Nobel prize for the unification of electromagnetic and weak force (electroweak force).
- It is expected that strong nuclear force will unite with electro weak force resulting in grand unified electroweak force.

Force	Nature	Range	Carrier particles	Relative strength	Effect
Gravitational	Attractive	Long range infinite (obey inverse square law)	Gravitone (not yet discover)	10^{-38}	➢ Binds the masses with each other such as stars and galaxies ➢ Responsible for binding the satellite, atmosphere and sea with the earth
Weak nuclear	Repulsive	Short range ($\sim 10^{-17}$m)	W^+, W^-, Z (Bosons)	10^{-13}	➢ Responsible for spontaneous breaking up of radioactive element. ➢ Responsible for β^+ and β^- decay
Electro magnetic	Attractive or repulsive	Long range or infinite (obey inverse square law)	Photons	10^{-2}	➢ Binds together atoms, molecules, crystal, trees ➢ Responsible for various macroscopic forces such as friction, adhesion, cohesion etc.
Strong nuclear	Attractive	Short range ($\approx 10^{-15}$m)	Gluons	1	➢ Effective only within subnuclear distances. ➢ Binds the neutrons and protons within the nucleus. ➢ Responsible for binding planets with the sun

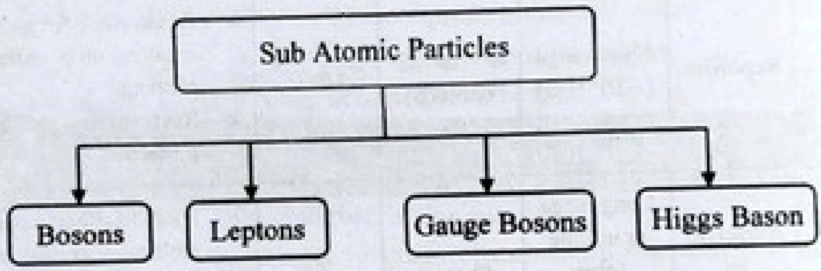
Quarks

- All photons quarks leptons are elementary particles.
- Hadrons are not elementary particles but are composed of quarks.
- According to quark theory by M. Gell Mann and G. Zweig the quarks are basic building block of mesons and baryons.
- It is proposed that there are six quarks

1. Up	2. Down	3. Strange
4. Charm	5. Bottom	6. Top

Six Types of Quarks		
Name	**Symbol**	**Charge**
Charm	c	$+\frac{2}{3}e$
Up	u	$+\frac{2}{3}e$
Top	t	$+\frac{2}{3}e$
Bottom	b	$-\frac{1}{3}e$
Down	d	$-\frac{1}{3}e$
Strange	s	$-\frac{1}{3}e$

Six Types of anti quarks		
Name	**Symbol**	**Charge**
Anti Charm	$\bar{c}$	$-\frac{2}{3}e$
Anti Up	$\bar{u}$	$-\frac{2}{3}e$
Anti Top	$\bar{t}$	$-\frac{2}{3}e$
Anti Bottom	$\bar{b}$	$+\frac{1}{3}e$
Anti Down	$\bar{d}$	$+\frac{1}{3}e$
Anti Strange	$\bar{s}$	$+\frac{1}{3}e$



Hadrons	***"Subatomic particles which experience strong nuclear force are called hadrons".*** Hadrons consist of quarks. **Example:** mesons and baryons **Mesons:** Subatomic particles having mass less than the proton are called mesons . Mesons consists of a pair of quark and anti quark. **Baryons:** Subatomic particles having mass greater or equal to proton are called baryons. Baryons consists of three quarks Proton and neutron are baryons
Leptons	***"Subatomic particles which don't experience strong nuclear force are called leptons".*** don't consist of quarks **Example:** electrons muons, tau and their associated neutrino
Gauge Bosons	***"Gauge Bosons are exchange particles or carriers of basic forces of nature".*** **Example:** photons are carriers of electromagnetic force • Gluons are carriers for strong nuclear force. • W^+, W^-, Z° Bosons are carrier of electro-weak force. • Gravitons are carrier of gravitational force
Higgs Boson	Higgs Bosons are discovered in July 2012, provides an explanation for how the other particles get mass by interacting with it.

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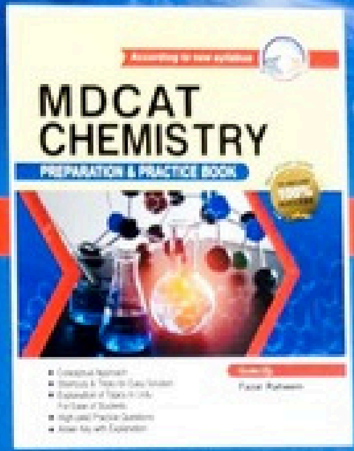

"I believe in ideology of smart work instead of hard work"

AZHAR IQBAL